

# An unsupervised 3D mesh segmentation based on HMRF-EM algorithm

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## ABSTRACT

We propose a new 3D mesh segmentation method based on the HMRF-EM framework. The clustering method relies on the curvature attribute and considers the spatial information encoded by the mutual influences of neighboring mesh elements. A region growing process is then carried out in order to extract connected regions followed by a merging procedure. The purpose of this latter process is to only preserve meaningful regions. Experiments conducted on different meshes are encouraging and show that the proposed method gives satisfying results compared with classical statistical ones such as kmeans and EM algorithms.

## Keywords

HMRF-EM algorithm, region growing, region merging, mesh segmentation.

## 1 INTRODUCTION

3D mesh segmentation has been an important 3D shape analysis topic, essential for a wide range of applications such as part-based shape recognition or retrieval, Sketch-based Shape Retrieval [Cha15], texture mapping, reverse engineering applications that deals with CAD models and component-shape based synthesis that provides new models by combinations of parts from existing models [Kal12].

3D mesh segmentation consists in partitioning the mesh into disjoint sub-meshes according to some specific criteria. Many segmentation algorithms have been proposed in the literature. They could be classified into part-type ones that decomposes the mesh into semantic and meaningful part and patch-type ones based on the mesh geometry attributes such as curvatures, convexity, roughness, etc.

Many 3D mesh segmentation methods have been proposed such as clustering ones, region-growing ones and spectral methods [Sha08]. In this work we focus on clustering methods that aim to associate an ap-

propriate cluster label to each mesh element according to some attribute values. Most of them are 3D extensions of well known 2D classification algorithms [Sha08, Lav08, Tsu14]. Curvature based descriptors are widely used attribute since curvature is a very significant criterion that describes the shape structure variation. However, clustering methods based on curvature attribute generate many isolated fragments as curvature is very sensitive to noise. So a post-treatment step is always needed to deal with such problem.

In [Lav08], a clustering method based on the Markov Random Fields (MRF) schema has been proposed. Authors initialise their proposed method by a K-means algorithm, the resulting labeled mesh is then median filtered and used for the prior and observation parameters estimation. The simulated annealing is subsequently applied for the resolution of the maximum a posteriori estimate (MAP) which is known to be a very time consuming algorithm. It's important to notice that few work dealt with the MRF extension to 3D mesh processing [And07, Wil04, Lav08].

In this work, we propose a new 3D mesh segmentation method based on the HMRF-EM clustering framework [Zan01]. This framework incorporates the HMRF model with the Estimation-Maximization (EM) algorithm. Unlike [Lav08], the iterated conditional mode is adopted for the optimization step that seeks the MAP estimate and for the prior model estimation an adaptive weighted cost function is also defined based on the dihedral angles of neighboring faces. This method

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takes into account both spatial and attribute information which yields to be robust to noisy data. In fact, the mesh geometry is encoded through the mutual influences of their neighboring sites. A post treatment based on a region growing method is then carried out in order to generate only connected components.

This reminder of the paper is organised as follows : in section 2, we describe the proposed method overview while section 3 details the HMRF-EM framework adapted for the 3D mesh dual graph. Section 4 deals with the post treatment that aims to extract connected regions from the resulting labeled mesh. Finally, some experiments and results on different meshes are shown in section 5.

## 2 METHOD OVERVIEW

Figure 1 shows the main steps of the proposed mesh segmentation method. First, the curvature attribute of the input triangular mesh, denoted by  $\mathcal{M}(V, F)$  where  $V$  is the set of vertices and  $F$  the set of triangles, is computed. Then, we carry out a facet-based clustering algorithm that combines HMRF model with EM algorithm. To deal with facet-based clustering we consider the dual graph  $\mathcal{M}^*$  of  $\mathcal{M}$  that is defined as follows : Each vertex of the dual graph corresponds to a triangle of  $\mathcal{M}$  and two vertices of  $\mathcal{M}^*$  are neighbors if and only if their corresponding triangles in  $\mathcal{M}$  share an edge. The curvature attribute associated to each facet gravity center is the mean curvature values computed on their vertices. It's important to notice that the proposed method could deal with many others attributes rather than curvature ones. Finally a connected region extraction step is performed in order to eliminate isolated parts.

## 3 THE HMRF-EM FRAMEWORK

### 3.1 Neighborhood and contextual relationship

In this work, we deal with the irregular dual graph  $\mathcal{M}^*(V^*, E^*)$  and we consider  $V^*$  as the set of sites denoted  $S$ . The MRF theory assumes that the sites are related to each other via a neighborhood system defined as  $\mathcal{N}_s = \{t \in S, t \neq s \text{ et } s \in \mathcal{N}_t\}$

In addition, a clique system is defined on  $S$  describing the configuration of all mutually neighboring sites or the site itself. In this work, we consider single-site clique and pair-site clique denoted respectively by  $C_1$  and  $C_2$  (see figure 2).

### 3.2 The Hidden Markov Random Field

Let  $X = \{X_s; s \in S\}$  and  $Y = \{Y_s; s \in S\}$  be two random fields corresponding respectively to labels and observations. The label field takes values in a discrete set  $L$  and the observation one in  $D$ .

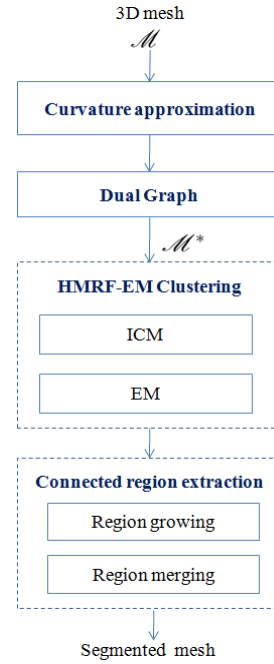


Figure 1: Method overview.

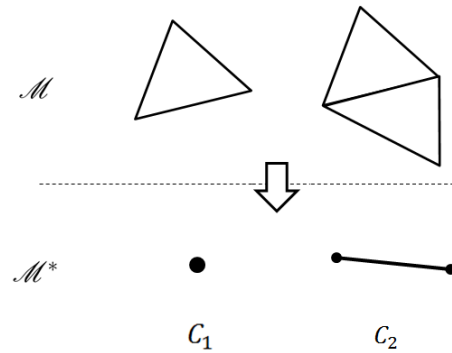


Figure 2: clique system.

In the segmentation context, we aim to estimate  $x$  a configuration of  $X$  based only on an observation  $y$  of  $Y$ . The underlying field  $X$  is non-observable, therefore the appropriate model is the Hidden Markov Random Field (HMRF).

A random field  $X$  is called a MRF on  $S$  with respect to the neighborhood system  $N$  if and only if  $P(x) > 0$  and  $P(x_s | x_{S-\{s\}}) = P(x_s | x_{\mathcal{N}_s})$ , where  $\mathcal{N}_s$  is the neighborhood of the site  $s$ . This last property expresses the behavior of the random variable on a site is determined by the neighboring random variables realisation and we can model practically all random variables whose mutual interdependence is resulting only from the combination of local interactions.

The Hammersley-Clifford's theorem [Bes74] establishes equivalence between MRF and Gibbs field. The distribution of  $X$  is given then by :

$$P(x) = \frac{1}{Z} \exp\left(\frac{-U(x)}{T}\right) \quad (1)$$

Where  $Z$  is the normalizing constant and  $U(x)$  the energy function which is the sum of clique potentials  $U_c(x)$  over all possible cliques  $C$ :

$$U(x) = \sum_{c \in C} U_c(x) \quad (2)$$

The energy  $U$  could be written as follows:

$$U(x_i = \ell | x_{\mathcal{N}_i}) = \sum_{j \in \mathcal{N}_i} \phi_{i,j} \delta(x_i, x_j) \quad (3)$$

Where  $\delta(i, j) = \begin{cases} -1 & \text{if } i = j \\ 1 & \text{else} \end{cases}$

and  $\phi_{i,j} = \|e_{ij}\| |\alpha_{ij}|$

Where  $\|e_{ij}\|$  is the length of the shared edge and  $|\alpha_{ij}|$  is the absolute of the angle between the normals of the two faces sharing an edge (figure 3) :

$$\alpha_{i,j} = \arccos \frac{n_{v_1 v_2 v_3} \cdot n_{v_2 v_4 v_3}}{\|n_{v_1 v_2 v_3}\| \|n_{v_2 v_4 v_3}\|} \quad (4)$$

Where  $n_{v_1 v_2 v_3}$  et  $n_{v_2 v_4 v_3}$  are the normals of the two faces and  $\cdot$  is the scalar product of vectors. The normal  $n_{v_1 v_2 v_3}$  is given by :

$$n_{v_1 v_2 v_3} = \frac{(v_2 - v_1) \times (v_3 - v_1)}{\|(v_2 - v_1) \times (v_3 - v_1)\|} \quad (5)$$

Where  $\times$  is the vector product of two vectors.

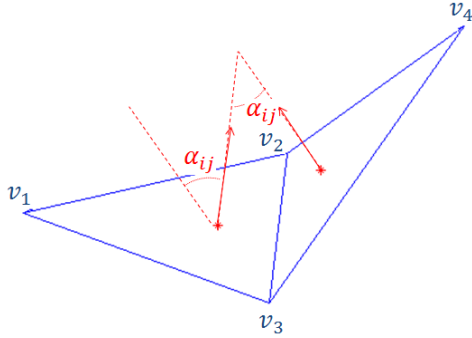


Figure 3: Dihedral angle: angle between the normals.

In a Bayesian context, we seek the solution of the maximum a posteriori expressing the most probable realization of the hidden variables given the observed one,

$$P(x|y) = \frac{P(x)P(y|x)}{P(y)} \quad (6)$$

Assuming that  $\{y_s\}_{s \in S}$  are conditionally independent

$$P(Y = y | X = x) = \prod_s P(Y_s = y_s | X_s = x_s) \quad (7)$$

The a posteriori probability became a function of the so called a posteriori energy

$$P(X = x | Y = y) \propto \exp(\text{Log}P(Y|X) - U(x)) \propto \exp(U(x|y)) \quad (8)$$

Where

$$U(x|y) = \sum_{s \in S} -\text{Log}P(y_s | x_s) + \sum_{c \in C} U_c(x) \quad (9)$$

And the maximum a posteriori estimator giving the labeling  $\hat{x}$  is equivalent to

$$\hat{x}_{MAP} = \text{argmin}_x (U(x|y)) \quad (10)$$

The observation model is a multi-variate Gaussian one. In this work, we consider a 2 dimensional observation (maximum and minimum curvature). Thus the a posteriori energy is given by :

$$U(x|y) = \sum_{s \in S} \left( -\frac{1}{2} (y_s - \mu_{x_s})^T \Sigma^{-1} (y_s - \mu_{x_s}) + \text{Log}(2\pi |\Sigma_{x_s}|^{1/2}) \right) + \beta \sum_{(s,t) \in C_2} \phi_{s,t} \delta(x_s, x_t) \quad (11)$$

where  $\mu_{x_s}$  and  $\Sigma_{x_s}$  are, respectively, the mean vector and the covariance matrix of class  $x_s$ .

To find the classification map  $\hat{x}$  by the maximum a posteriori estimator corresponding to a minimization of the energy function  $U(X|Y)$ , a global optimization algorithm can be used such as simulated annealing. This algorithm was initiated by Kirkpatrick [Kir84] and adopted by Geman and Geman [Gem84] in the image processing context. The simulated annealing aims to find the global minimum of the energy that may have several local minimum. With analogy to thermodynamics, this algorithm incorporates a decreasing temperature parameter into the minimization procedure. For each temperature, it iteratively updates the energy function that will be accepted or rejected according to its probability which depends on the temperature parameter. This process is repeated until equilibrium state is reached. The simulated annealing algorithm ensures convergence to a global minimum energy but generates a large number of configurations as the temperature decreases which makes it a very time consuming algorithm. To overcome this disadvantage, we often use local algorithms such as the iterated conditional modes (ICM). This algorithm was proposed by Besag [Bes86], its principle is to iteratively update the sites labels based on the observation  $y$  and the current neighbors configuration of the each site. The new value  $\hat{x}_s$  is obtained by maximizing the local probability  $P(x_s | x_{\mathcal{N}_s}, y)$ .

Since we are in a parametric context, an estimation of the Gaussian distribution parameters is required. In this work, we use an unsupervised algorithm of the maximum likelihood, the Expectation- Maximization (EM) algorithm.

### 3.3 The HMRF-EM algorithm

The combination of the ICM algorithm aiming to estimate the MAP resulting from the MRF theory and the EM algorithm gives an iterative algorithm called HMRF-EM and it can be resumed as follows:

In an iteration  $t$ :

- estimation of  $\hat{x}$  by ICM
- estimation of  $\theta_\ell(\mu_\ell, \Sigma_\ell)$

$$\hat{\mu}_\ell^{(t)} = \frac{\sum_{i \in S} P^{(t)}(\ell|y_i) y_i}{\sum_{i \in S} P^{(t)}(\ell|y_i)} \quad (12)$$

$$\hat{\Sigma}_\ell^{(t)} = \frac{\sum_{i \in S} P^{(t)}(\ell|y_i) (y_i - \hat{\mu}_\ell^{(t)})^T (y_i - \hat{\mu}_\ell^{(t)})}{\sum_{i \in S} P^{(t)}(\ell|y_i)} \quad (13)$$

where

$$P^{(t)}(\ell|y_i) = \frac{P^{(t)}(y_i|x_\ell, \theta_\ell) P^{(t)}(\ell|\hat{x}_{\mathcal{N}_i})}{P(y_i)} \quad (14)$$

The spatial information is encoded in the prior distribution  $P^{(t)}(\ell|\hat{x}_{\mathcal{N}_i})$  given by :

$$P(\ell|\hat{x}_{\mathcal{N}_i}) = \frac{\exp(-U(\ell|V_i))}{\sum_{\xi \in L} \exp(-U(\xi|V_i))} \quad (15)$$

## 4 CONNECTED REGION EXTRACTION

Once faces have been classified, a labeling operation is performed in order to extract connected significant regions. This procedure consists of a region growing step that produces a large set of connected regions which will be reduced with the following merging step (figure 4). In fact this latter one aims to merge similar neighbor regions according to a region distance measure [Lav05, Lav04]. In what follows, we briefly describe the region growing-region merging procedure.

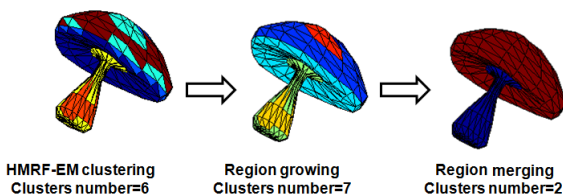


Figure 4: The region growing-merging process.

### 4.1 Region growing

Starting from seed triangles corresponding to faces which its neighbors belong to the same cluster, we iteratively expand the regions with new labels (each identified seed triangle is considered as a new region). Each triangle  $T_i$  that it is not yet labeled and has the same cluster as the seed triangle of the region

that aggregate its neighbors joins this latter region. The growing step normally leads to holes between the identified connected regions (the triangles in the boundary of two regions are not labeled). In order to fulfill those holes, we assign a not labeled triangle to the most represented region in its neighbors and we repeat this process until every triangle is labeled. This step is called crack filling.

### 4.2 Region merging

The number of connected region produced by the growing step depends on the number of the clusters from the faces classification performed by the HMRF-EM algorithm. Generally, numerous small regions are identified and need to be merged with similar ones in order to have significant areas and for that a region adjacency graph (RAG) is used. The nodes on this graph represent the connected regions and the edges represent an adjacency between two regions. Edges are weighted with a similarity distance between the two corresponding regions. The region distance measure  $D_{ij}$  between two adjacent regions  $R_i$  and  $R_j$  is given by:

$$D_{i,j} = DC_{ij} \times N_{ij} \times S_{ij} \quad (16)$$

Where  $DC_{ij}$  is the curvature distance between  $R_i$  and  $R_j$  and equal to  $\|C_i - C_{ij}\| + \|C_j - C_{ij}\|$ ,  $C_i$  and  $C_j$  are respectively the curvature values of  $R_i$  and  $R_j$  corresponding to the mean of the faces curvature of each region, and  $C_{ij}$  is the mean curvature of vertices on the boundary between  $R_i$  and  $R_j$ . The  $N_{ij}$  coefficient measures the nesting between the two corresponding regions which describes the spatial disposition of the regions and the  $S_{ij}$  coefficient allows to accelerate the merging of the smallest regions.

The processing of the graph reduction is as follows: at each iteration, the edge that has the smallest weight is eliminated and hence the corresponding regions are merged. Since the regions number is decreased by one, the graph is then updated and the process is repeated until the weight of the smallest edge is larger than a given threshold or a fixed number of regions is reached.

## 5 EXPERIMENTAL RESULTS

In figure 5, we compare our segmentation method with the Kmeans and EM clustering algorithm using 4 objects. In this experiment we set the number of clusters to 2 for the octopus and the dinosaur objects and to 4 for the vase and the eyeglass objects. Similar results are obtained for the octopus object identifying 9 regions (the head and the 8 arms). For the vase object the kmeans algorithm seems to provide finer decomposition than the EM and the HMRF-EM algorithm. In fact it allows to distinguish 6 regions rather than 3. We

can note that both partitions (3 or 6 region decomposition) correspond to meaningful parts of the object. Considering the eyeglass object, our method overperforms the kmeans and the EM algorithm since it enables to extract the two temples and the frame. In the case of the dinosaur object, the head was extracted only by the HMRF-EM method.

In order to show the efficiency of the proposed method for meshes that presents different curvature variations, we conducted experiments for 6 models (see figure 6). Our method provides good results for objects presenting low and medium curvature changes (first row in Figure 6). It generates non meaningful regions otherwise. In table 1, we measured the computational time for meshes presented in figure 6. The most time consuming step of the HMRF-EM algorithm is the classification step by the ICM algorithm where the prior energy is computed using neighboring triangles. Its complexity is equal to  $O(\ell \times K \times N) = O(N)$ , with  $\ell$  is the upper bounds of the iterations number,  $K$  is the clusters number and  $N$  is the triangles number. Thus the complexity of the HMRF-EM algorithm is linearly dependent of the triangles number to be classified. As we can clearly notice in the table 1, for too dense meshes, the computation time is much higher than the one for simple meshes. The HMRF-EM algorithm was implemented with MATLAB. All experiments were performed on a PC with an Intel Core i7, CPU 2.5GHz and 8GB RAM.

3D Model	N	K	Processing time(s)
Mushroom	448	8	3.87
Octopus	2682	4	11.24
Bearing	7227	7	54.11
Fish	15142	4	71.31
Cup	30254	6	227.35
Bust	50456	10	645.25

Table 1: The computing time for different meshes

## 6 CONCLUSION

In this paper, a new 3D mesh segmentation based on the HMRF-EM framework has been proposed. The markov random field modelization is combined with the EM algorithm for parameters estimation. The definition of the prior model for this extended algorithm is based on dihedral angles which favorise the grouping of triangles having similar curvature values. After this clustering step, a region growing-merging process is applied in order to extract connected regions. Results show the efficiency of this extended framework. In a future work, we propose to use different type of attributes such as local mesh descriptors to improve segmentation results for more complex objects. In particular we aim to consider the 3D spectral information where the low frequencies correspond to the global shape while the hight frequencies contribute to the geometric details.

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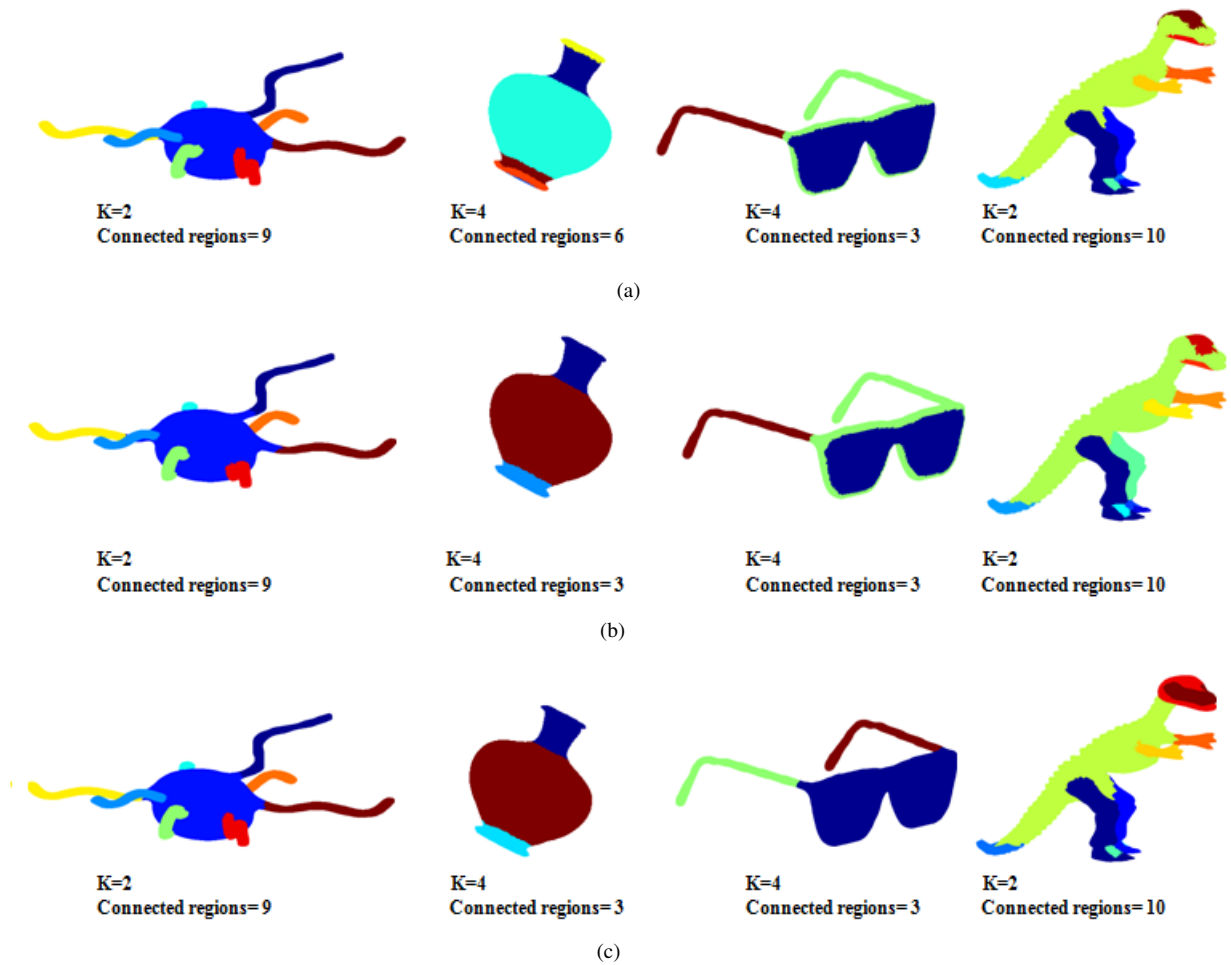


Figure 5: (a) K-Means and region growing, (b) the 2-dimensionnal EM and region growing, (c) the HMRf-EM and region growing.

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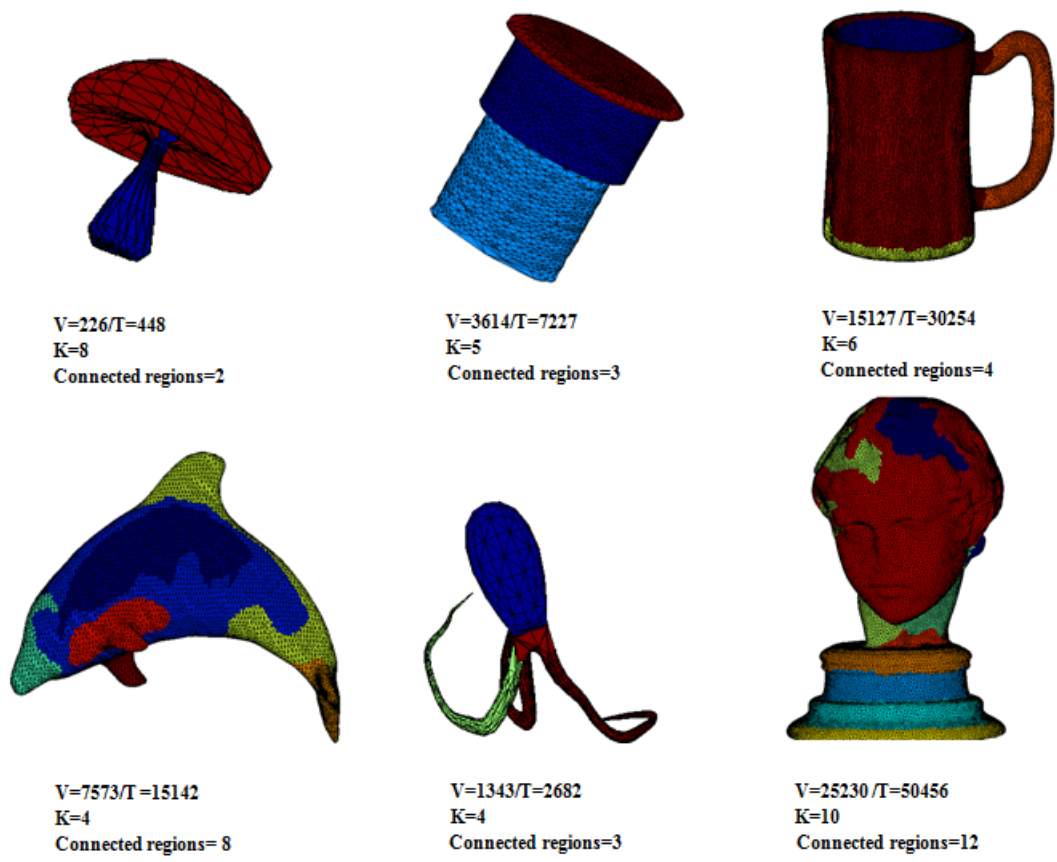


Figure 6: Segmentation results for objects with different curvature variations.