Search for small monostatic polyhedra

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ABSTRACT

We study a class of convex polyhedra that contains the famous 19-face monostatic polyhedron discovered by R. K. Guy in 1969. The properties of the polyhedra of this class allow to efficiently scan them so that it becomes possible to check in a reasonable time, with a computer program, whether the class contains a convex monostatic polyhedron with fewer than 19 faces.

Keywords

Monostatic, polyhedron.

1. INTRODUCTION

A convex monostatic polyhedron (CMP) is an homogeneous convex polyhedron that remains in stable equilibrium on only one of its faces. If laid on any other face, it rolls until it finally stops on the only face on which it is stable. In 1969, R. K. Guy and, two months later, Ken Knowlton independently discovered such a solid that had only nineteen faces (Figure 1) [Guy69] [Bry08].

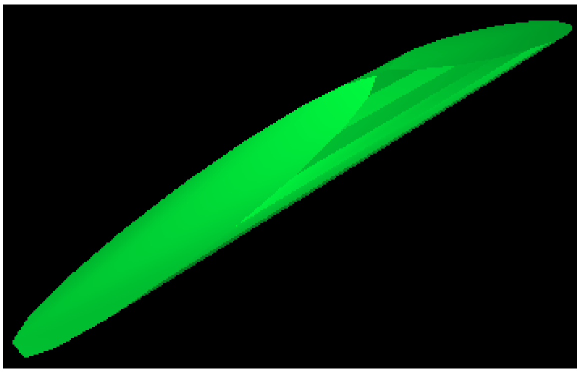

Figure 1: Guy's monostatic polyhedron

In dimensions other than 3, or for some special categories of polytopes, there has been some other results. For example, Arnold has proven that no 2-polytope (2D convex polygon) can be monostatic. In other words, any convex polygon is stable on at least two of its edges; another proof is available in [Pak].

Conway has also proven that there are no monostatic simplices in dimension 3 (no reference available) and Dawson did the same for dimensions up to 6 [Daw85]. This result has further been extended to dimensions 7 and 8 in [DFM98]. Surprisingly, Dawson has also proven that there exists monostatic simplices in dimension 10 and up [DaF01].

In dimension 3, there is no theoretical value for the least number of faces of a monostatic convex polyhedron. Up to last year, 19 was the minimum known value but, in last September, Andras Bezdek presented a construction procedure to build a 18-face CMP (Figure 2) [Bez11].

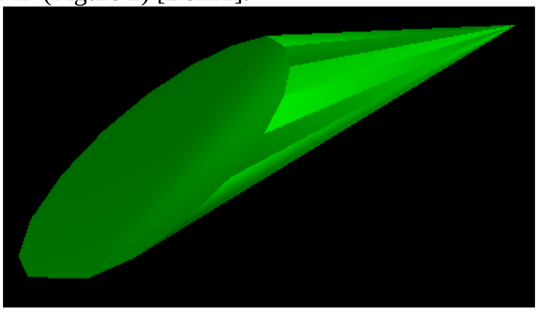

Figure 2: Bezdek's monostatic polyhedron

Besides this, Guy has proven that the least number of faces for the CMPs built with his procedure is 19 but, if some of the constraints he has combined to produce his CMP are relaxed, one still gets CMPs (see section 2). Because of this, and because it is now known that 19 is not a minimum, we think it is worth trying to find CMPs similar to Guy's object but with fewer than 19 faces.

This paper summarizes an experiment conducted in this goal: it has consisted in exploring a class of polyhedra similar to Guy's one with kind of an exhaustive traversal (with a given step). It has not been successful, in the sense that many CMP with 19 faces have been found but none with less than 19. On the other hand, a 17-face solid that is very close from being a CMP has been found in the class, so that some doubts remain on the fact that this class contains a CMP with fewer than 19 faces.

The paper is structured as follows: parts 2 and 3 present Guy's CMP and a class of polyhedra to which it belongs. Part 4 gives an algorithm to scan all the polyhedra in this class and part 5 details the first optimization of this algorithm. The next part gives and proves an important property concerning the center of mass of the studied polyhedra and part 7 explains how to use this property to implement another optimization. Then a method to evaluate the quality of a polyhedron of the class is described (is it monostatic or, if it is not, how far is it from being monostatic). The last part gives some results, concluding remarks and presents planned future work.

2. GUY'S OBJECT

For an convex polyhedron to be in stable equilibrium when laid on one of its faces, its center of mass has to project within this face. For example, the tetrahedron below (Figure 3) [Hep67] is in stable equilibrium on faces (A,B,D) and (A,C,D) only (and also has the unusual property that, when laid on one of the other two faces, it first rolls on the other unstable face before rolling again on a stable face). So, for a convex polyhedron to be monostatic, its center of mass must project in one and only one face.

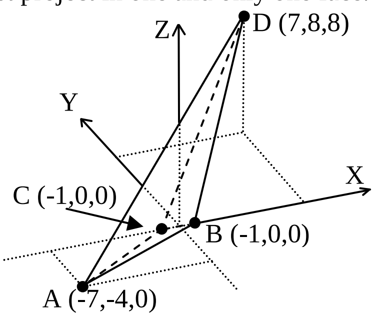

Figure 3: a tetrahedron with two stable faces

Guy's solid is a prism truncated by two symmetric planes. The section of the prism is a polygon of the xz-plane, symmetric with respect to the z- axis; the half-section is obtained by gluing a sequence of m right triangles with the same aperture π/m (Figure 4). As the bottom edge and its symmetric generate the same face, the total number of faces is $1 + 2.(m-1) + 2 = 2.m + 1$.

Guy proved that, if $m \geq 9$, by tilting and moving away the truncation planes sufficiently, it is possible to lower the center of mass so that:

- it stands lower than the point shared by all the triangles (O in Figure 4); so, it projects only in the lowest lateral face;

- it projects in none of the "caps" (the faces generated by the truncation).

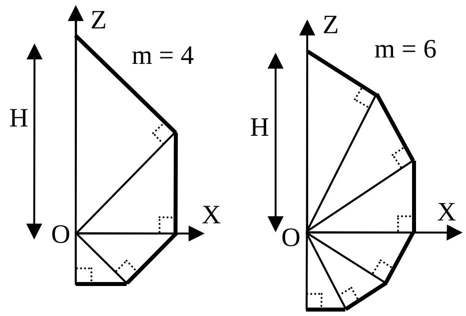

Figure 4: the half-sections of two Guy's objects

So the least number of faces for a CMP built with this procedure is 19.

But if one slightly moves some vertices of the profile of such a CMP (so the new profile no longer meets Guy's requirements), it is still possible to build a CMP. For example, if H = 100 (see Figure 4) and m = 9, the vertices of the half-profile of Guy's object are those in Figure 5-left; but with the vertices of Figure 5-right (this half-profile is also made of 9 right triangles but their apertures are not all the same) and with cutting planes $y = -6.z + 624$ and $y = 6.z - 624$, we still get a CMP.

Guy's half-profile		Another valid half-profile	
X	Z	X	Z
0	100	0	100, 581019047
32.139380	88.302222	32. 326114341	88, 815276618
56.759574	67.643427	58, 637925590	65, 124016131
71.860144	41.488473	72, 895434578	37, 142079636
76.788242	13.539839	77, 297886249	16, 430172842
72.157344	-12.723287	72.157344678	-12.723286842
59.627353	-34.425868	59.627353253	-34.425868547
41.588017	-49.562669	41.588017084	-49.562668849
20.794008	-57.131069	20.794008542	-57.131069
0	-57.131069	0	-57.131069

Figure 5: two valid 19-face half-profiles.

Hence, Guy's construction procedure is not the only one that leads to half-profiles that allow to build CMPs; and may be the least number of faces of CMPs built with other procedures is smaller than 19. That's why we have undertaken to explore several classes of polyhedra; the class studied in this paper in described in the next section.

3. THE CLASS OF POLYHEDRA STUDIED IN THIS PAPER

The polyhedra studied in this paper have the following properties (Figure 6):

- they are prisms whose main axis is parallel to the y-axis, truncated by two planes that are symmetric with respect to the xz-plane;
- the profiles of the prisms are convex polygons of the xz-plane, symmetric with respect to the z-axis;

- a half-profile always starts with a line segment whose slope is negative and ends with a line segment perpendicular to the z-axis. Without loss of generality, the ends of the half-profile can be fixed to (0,0,100) and (0,0,0);
- the slopes of the cutting-planes are such that the faces they generate, the "caps", have an upwards outward normal vector (the z-coordinate of the outward normal vector is strictly positive);
- the cutting-planes are perpendicular to the yz-plane, so that the x-coordinate of the normal vector to the caps is 0.

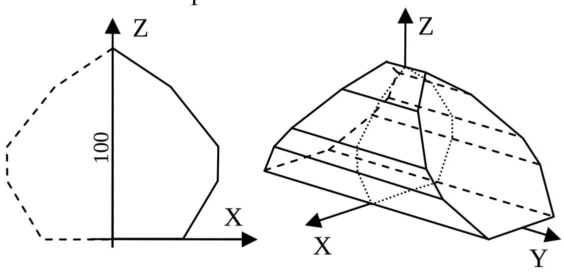

Figure 6

In one word, the polyhedra in this class have the same aspect as Guy's object, except that the half-profile just has to be convex. For any polyhedron in this class, due to the symmetries, the center of mass G lies on the z-axis , somewhere between (0,0,0) and (0,0,100) and the base face, generated by the lowest segment of the half-profile and its symmetric, is a stable face, as G projects in it. So, for a element of the class to be monostatic, G must project in no other face. This implies that G must project on no other edge of the profile but the base edge.

Note that Guy's object belongs to the class and that, for any polyhedron in the class, if its half-profile contains m edges, it has 2.(m-1)+3 = 2.m+1 faces. For a CMP to have fewer faces than Guy's object, m may vary from 2 to 8.

4. SCANNING THE CLASS

The base mechanism to traverse all the polyhedra of the class for a given m is to generate all possible m-sided half-profiles and, for a given half-profile, to enumerate all possible cutting planes. Each produced polyhedron is then checked for monostaticity. Letting m grow from 2 to 8 achieves a complete scan of the polyhedra we wish to study. In fact, we'll see later in the paper that it is useless to enumerate the cutting planes and to really build the polyhedra : an efficient validity test may be performed as soon as the half-profile is available. If the half-profile is declared to be valid, in a sense given below, it is sure that it is possible to exhibit cutting planes that lead to a monostatic polyhedron.

Of course, the generation of all possible half-profiles is performed with a given step (in fact, there are an angular step, $\Delta\alpha$, and a distance step, ΔL); the thinner the steps, the slower the algorithm. These steps are input parameters.

We first present the m-sided half-profile generation process and then its optimizations. This process is recursive. Let k be the current number of vertices in the profile (k=0 at the beginning as the profile is empty) and let P_i be the i^{th} vertex of the half-profile ($i \geq 1$):

Case 1: k = 0 (the profile is empty)

Add vertex P_1 (0,0,100), set k to 1 and recursively complete the half-profile

---

Case 2: 0 < k < m-1

For the profile to remain convex, the angle for the next edge $[P_k, P_{k+1}]$ may vary from α_{min} to α_{max} (both excluded), where :

- α_{min} is the anti-clockwise angle of $[P_k , O]$ with the horizontal (Figure 7)
- α_{max} is the anti-clockwise angle of $[P_{k-1} , P_k]$ with the horizontal

Exception: if k = 1, $\alpha_{min} = 3.\pi/2$ and $\alpha_{max} = 2.\pi$.

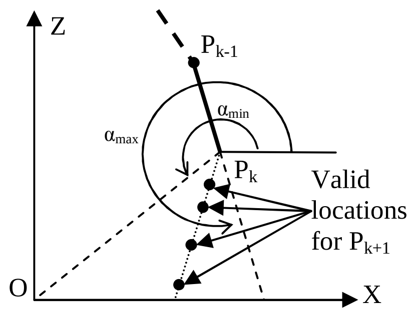

Figure 7: angular interval (dashed lines) and distance interval for the next vertex

For a given angle, the length of the next side may vary from 0 (excluded) to the distance D to the x-axis (excluded) (Figure 7). Beyond this length, the profile can't remain convex once O is added. So the algorithm is:

---

for α from α_{min} to α_{max} (both excluded) step $\Delta\alpha$

- compute the intersection I between the x-axis and the line of angle α starting in P_k. Let D be the distance from P_k to I.
- for d from 0 to D (both excluded) step ΔL
  add P_k + d.(cos (α), 0, sin (α)) to the half-profile, add one to k and recursively complete the half-profile.

Case 3: k = m-1 (two more vertices must be added: P_m and P_{m+1})

The next vertex, P_m, has to be on the x-axis (Figure 8).

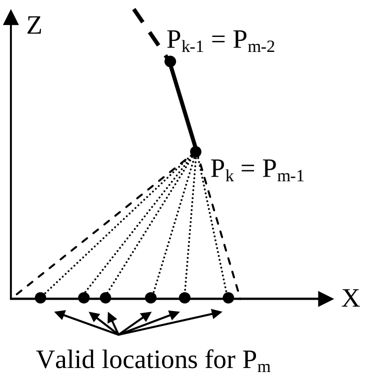

Figure 8: choosing the penultimate vertex

For the profile to remain convex, the angle for the next edge $[P_k, P_{k+1}]$ (that is $[P_{m-1}, P_m]$) may vary from α_{min} to α_{max} (both excluded and defined as above). So the algorithm is:

---

for α from α_{min} to α_{max} (both excluded) step Δ α

- compute the intersection of the x-axis with the line of angle α starting in P_k.
- add this point to the half-profile, set k to m and recursively complete the half-profile.

Case 4: k = m (the half-profile just lacks the last vertex)

Add vertex P_{m+1} (0,0,0), set k to m+1 and test the half-profile (see section 8)

Figure 9 shows some of the half-profiles produced with this method for m = 5.

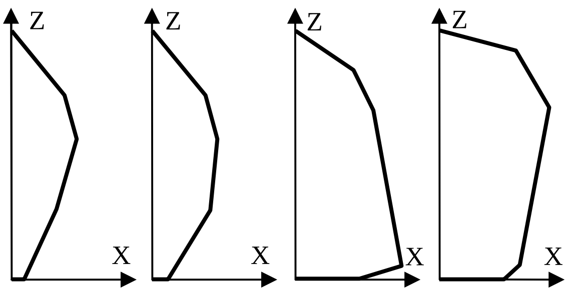

Figure 9: some half-profiles generated by the algorithm

We have no formula to express the time complexity of the half-profile generation but a simple reasoning gives a good idea of the unreasonable duration of the basic version. The iterative version of the algorithm is a sequence of 2.m - 3 nested loops:

...
for α_1 from α_{min1} to α_{max1}
  for d_1 from 0 to D_1
    add P_2 to the half-profile
    for α_2 from α_{min2} to α_{max2}
      for d_2 from 0 to D_2
        add P_3 to the half-profile
        ...
          for α_{m-1} from α_{min_m-1} to α_{max_m-1}
            add P_m, P_{m+1}
            test the half-profile

Assume Δ α and Δ L both equal one. Then α_1 can take 89 values (from 270° to 360° both excluded); if α_1 is small (280 degrees, for example) then α_2 can only take a few values but if α_1 is large (350°), α_2 can vary up to 350°. Depending of d_1, this may represent more than 100 values. To make a rough approximation, we suppose that all loops (α and d loops) iterate only 20 times and that each instruction lasts 10^{-9} seconds. Then, for m = 8, only for the half-profiles generation, we get a duration of:

$$20^{2.m-3}.10^{-9} \text{ s} = 948 \text{ days}$$

If variations of the cutting planes, calculation and evaluation of the resulting polyhedra had to be added, it is clear that a computational exploration would be unaffordable.

5. FIRST OPTIMIZATION

Let $PP_{i\ (1 < i \leq m)}$ be the intersection point between the z-axis and the normal to $[P_{i-1}, P_i]$ in P_i (PP_i stands for Projected P_i) (Figure 10.a).

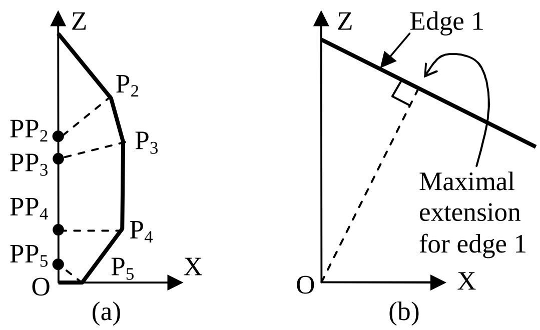

Figure 10

If PP_2 is lower than O, any point in $[P_1, O]$ projects on the first edge of the profile, including the center of mass as it is somewhere in $[P_1, O]$ by construction. So it is certain that the profile cannot lead to monostatic polyhedra. The consequence for the algorithm is that, once the slope of the first edge is chosen, it is useless to extend the first edge beyond the projection of O on this edge (Figure 10.b).

This optimization can be applied to any of the first m-2 edges but it is useful only for the edges whose angle is larger than $3.\pi/2$. Below this limit, the x-axis is intersected before the projection of O is reached. So case 2 of the algorithm becomes:

Case 2: $0 < k < m-1$

```
for α from α_min to α_max (both excluded) step Δ α
    if α > 3. π/2
        D = distance from P_k to the projection of O
        on the line of angle α starting at P_k
    else
        compute the intersection I between the x-
        axis and the line of angle α starting in P_k.
        D = distance from P_k to I

    for d from 0 to D (both excluded) step Δ L
        add P_k + d.(cos (α), 0, sin (α)) to the half-
        profile, add one to k and recursively
        complete the half-profile.
```

Case 3 is modified similarly. With this optimization, half-profiles 3 and 4 in Figure 9 are no longer generated as edge 3 (number 3) or edge 1 (number 4) are too long.

6. LOWER BOUND FOR THE Z-COORDINATE OF THE CENTER OF MASS

As depicted in Figure 11.a, the polyhedra of this study are composed of three parts: a prism (central part) and two truncated prisms (at both ends). For a given half-profile, the prism, the profile and the half-profile have centers of mass with the same z-coordinate, we call it ZG_1. The centers of mass of the two truncated prisms have the same z-coordinate, called ZG_2 (Figure 11.b). This section mainly aims at proving that whatever the slope of the cutting planes is, ZG_2 remains the same: the z-coordinate of a truncated prism is *not* a function of the slope of the cutting plane.

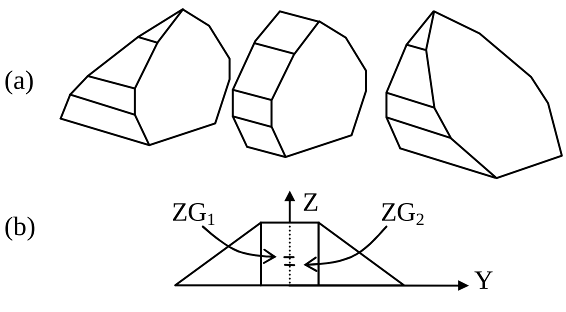

Figure 11

To do this, without loss of generality, we suppose that the truncated prism is positioned as depicted in

Figure 12.a. In particular, the cutting plane contains P_1 so its equation is $y = a.z + b$ where $b > 0$ fixes its slope and where $a = -b / z_1$ (Figure 12.a). Besides this, we only consider the truncated prism produced by the half-profile as its center of mass has the same z-coordinate as the center of mass of the truncated prism. The half-profile is divided into (m-1) triangles $(P_1, P_i, P_{i+1})_{1 < i \leq m}$ and the truncated prism it generates is divided into (m-1) truncated triangular prisms built from these triangles (Figure 12.b).

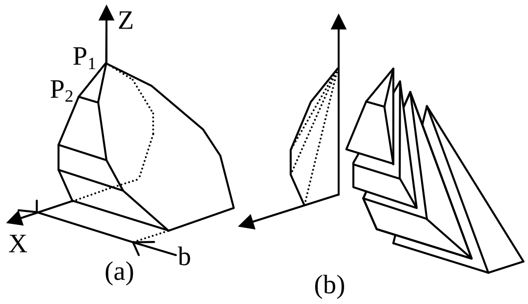

Figure 12: partition of the truncated prism

Let us first calculate the volume and the z-coordinate of the center of mass of the i[th] truncated triangular prism generated from the i[th] triangle. For the sake of clarity, the vertices of the triangle are not denoted (P_1, P_i, P_{i+1}) but (A,B,C), where $A=(x_A,0,z_A)$, $B=(x_B,0,z_B)$ and $C=(x_C,0,z_C)$. Without loss of generality, we can assume that $z_A \leq z_B \leq z_C$ and, at the moment, we assume that $z_A < z_B < z_C$ (Figure 13.a).

The equations of lines (A,B), (A,C) and (B,C) are:

$x = \alpha_{AB}.z + \beta_{AB}$ where $\alpha_{AB} = (x_B - x_A) / (z_B - z_A)$ and $\beta_{AB} = x_A - z_A.\alpha_{AB}$

$x = \alpha_{AC}.z + \beta_{AC}$ where $\alpha_{AC} = (x_C - x_A) / (z_C - z_A)$ and $\beta_{AC} = x_A - z_A.\alpha_{AC}$

$x = \alpha_{BC}.z + \beta_{BC}$ where $\alpha_{BC} = (x_C - x_B) / (z_C - z_B)$ and $\beta_{BC} = x_B - z_B.\alpha_{BC}$

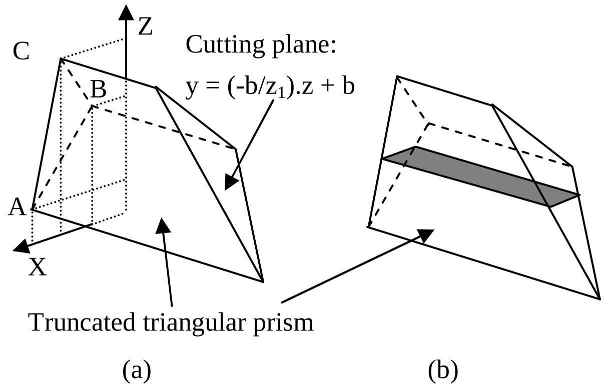

Figure 13

The intersection between the triangle and a line of the xz-plane, parallel to the x-axis, located between A and B and whose z-coordinate is z is a line segment of length $| \alpha_{AC}.z + \beta_{AC} - \alpha_{AB}.z - \beta_{AB} |$. This line segment generates in the prism a rectangle of area $(a.z + b) . | \alpha_{AC}.z + \beta_{AC} - \alpha_{AB}.z - \beta_{AB} |$ (Figure 13.b) and, of course, the z-coordinate of the center of mass of this rectangle is z. If the line is between B and C, the formulas are similar. So the volume V_i of the truncated triangular prism is the sum of the areas of all these rectangles, for z varying first from z_A to z_B and then for z varying from z_B to z_C:

$$V_i = \int_{z=z_A}^{z=z_B} (a.z+b).\left|\alpha_{AC}.z+\beta_{AC}-\alpha_{AB}.z-\beta_{AB}\right|.dz + \int_{z=z_B}^{z=z_C} (a.z+b).\left|\alpha_{AC}.z+\beta_{AC}-\alpha_{BC}.z-\beta_{BC}\right|.dz$$

As a = -b / z_1, V_i can be factorized under the following form:

$$V_i = b.\left(\int_{z=z_A}^{z=z_B}\left(\frac{-z}{z_1}+1\right).|\dots|.dz + \int_{z=z_B}^{z=z_C}\left(\frac{-z}{z_1}+1\right).|\dots|.dz\right)$$

that is: $V_i = b.M_i$ where M_i contains no occurrence of b.

The center of mass is the weighted average of the centers of mass of the rectangles, so its z-coordinate ZG_i is:

$$z_{G_i} = \frac{1}{V_i}\int_{z=z_A}^{z=z_B} (a.z+b).\left|\alpha_{AC}.z+\beta_{AC}-\alpha_{AB}.z-\beta_{AB}\right|.z.dz + \frac{1}{V_i}\int_{z=z_B}^{z=z_C} (a.z+b).\left|\alpha_{AC}.z+\beta_{AC}-\alpha_{BC}.z-\beta_{BC}\right|.z.dz$$

Again, b can be put into factor and ZG_i can be put in the following form:

$$z_{G_i} = \frac{1}{V_i}.b.R_i$$

where R_i contains no occurrence of b.
In other words,

$$z_{G_i} = \frac{R_i}{M_i}$$

where neither R_i nor M_i contain any occurrence of b.

If two points of the triangle have the same z-coordinate (the assumption $z_A < z_B < z_C$ becomes wrong), one of the integrals vanishes, in both R_i and M_i, but z_{G_i} keeps the same aspect (a ratio with no occurrence of b).

We can now calculate the z-coordinate of the center of mass of the truncated prism: it is the weighted average of the z-coordinates of the centers of mass of the (m-1) truncated triangular prisms:

$$ZG_2 = \frac{\sum_{i=2}^{m} V_i \,.\, z_{G_i}}{\sum_{i=2}^{m} V_i} = \frac{b.\sum_{i=2}^{m} R_i}{b.\sum_{i=2}^{m} M_i} = \frac{\sum_{i=2}^{m} R_i}{\sum_{i=2}^{m} M_i}$$

So b, which controls the slope of the cutting plane, does not appear in the expression of ZG_2. This ends the proof that ZG_2 does not depend on the slope of the cutting plane but only on the shape of the profile.

This has interesting consequences for the polyhedra of this study. The z-coordinate of their center of mass is:

$$ZG = \frac{VP}{VP+2.VTP}.ZG_1 + \frac{2.VTP}{VP+2.VTP}.ZG_2$$

where VP is the volume of the prismatic part and VTP is the volume of the truncated prism. This means that the minimum z-coordinate of the center of mass of all the polyhedra that can be built from a given half-profile is ZMinG = Min (ZG_1, ZG_2). Besides this, if $ZG_1 > ZG_2$ (ZMinG = ZG_2), by increasing the slope of the cutting planes, that is by getting them closer to the horizontal, it is possible to increase VTP while VP remains the same. This means that the center of mass can be brought infinitely close to (0, 0, ZG_2), as the weight of ZG_1 in the previous formula tends to 0 and the one of ZG_2 tends to 1. Symmetrically, if $ZG_1 < ZG_2$, by bringing the cutting planes closer and closer to the vertical and by moving them away, the weight of ZG_1 increases and the center of mass can be brought infinitely close to (0, 0, ZG_1). In other words, for a given profile, by acting on the cutting planes, it is possible to build a polyhedron whose center of mass is infinitely close to the lower of the two partial centers of mass (the one of the prism and the one of the truncated prism).

7. SECOND OPTIMIZATION

Remind that PP_i is the projection on the z-axis of the i^{th} vertex of the profile (P_i), perpendicularly to the (i-1)th edge. For a given half-profile (under construction or complete), we call "forbidden zone" the line segment from P_1 to the lowest PP_i (Figure 14.a). This zone can easily be incrementally maintained each time a point is added to the half-profile: one just has to project the new vertex on the z-axis and to compare the z-coordinate of the projected point with the current lowest point of the forbidden zone; the forbidden zone is thus available at any stage of the construction of a half-profile. We call ZMinFZ the z-coordinate of the lowest PP_i. The forbidden zone has two properties useful to the second optimization: first, any point in the forbidden zone projects on at least one of the edges of the half-profile under construction; second, the forbidden zone can never reduce: it consists in a single point when the half-profile is added its first point and increases downwards or remains the same each time a vertex is added to the half-profile.
Consider now a half-profile under construction; among all the manners to complete it (Figure 14.b), the one that allows to build the polyhedron with the lowest center of mass is the one where as much

matter as possible is added in the lower part of the profile. Let's call PHP this particular half-profile. PHP is obtained by extending the current last edge of the half-profile up to its intersection with the x-axis (in dashed line in Figure 14.b) and then by adding O.

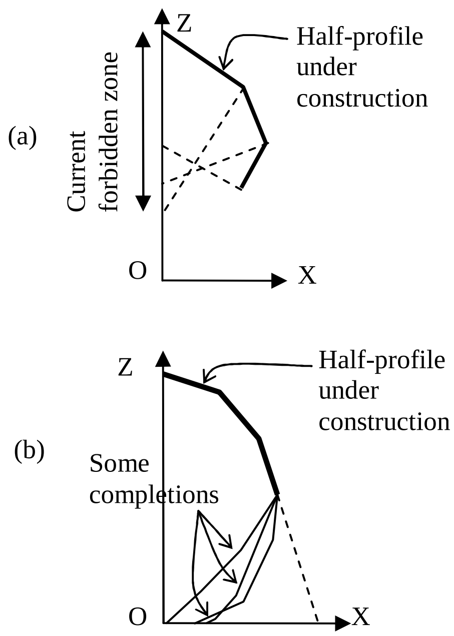

Figure 14

We can (see previous section) calculate the lowest possible z-coordinate for the centers of mass of all polyhedra that would be built from PHP, we call it ZMinG. By construction, we know that the centers of mass of all the polyhedra that can be built from the half-profile under construction are above ZMinG. If ZMinG > ZMinFZ, we can deduce that all these centers of mass are in the forbidden zone so they project in one of the side faces. As the forbidden zone cannot reduce while the half-profile is completed, we know that none of these polyhedra can be monostatic so it is useless to complete the current half-profile. So the second optimization just consists in testing, each time a vertex is added to a half-profile under construction, whether ZMinG > ZMinFZ. If this occurs, it is useless to complete the current half-profile and a huge amount of calculations is saved.

8. VALIDATING A PROFILE

The algorithm presented in part 4 and the optimizations of part 5 and 7 allow to enumerate potentially interesting half-profiles. Each time a half-profile is completed, it must be checked whether it can produce a CMP. For this, the test is strictly the same as the one of the second optimization: ZMinFZ and ZMinG (which are now those of a complete half-profile) are compared. If ZMinG ≥ ZMinFZ, it is sure that no CMP can be built from the half-profile. Otherwise, we know it is possible to build a polyhedron whose center of mass, G, is infinitely close to (0,0,ZMinG) (section 6). We know that G doesn't project on any edge of the profile (except the last one, of course). The remaining problem is that G should not project in one of the two oblique faces. This can be achieved easily: if ZMinG=ZG2, it suffices to increase the slope of the cutting planes until G doesn't project in the oblique faces anymore; this increases the relative volume of the truncated prisms and gets G even closer to ZMinG (which ensures that it doesn't project in any side face except the base face). If ZMinG=ZG1, it suffices to extend the prismatic part of the polyhedron without changing the slope of the planes, until G doesn't project in the oblique faces anymore; this increases the relative volume of the prismatic part and brings G even closer to ZMinG. So validating a profile just consists in checking that ZMinG < ZMinFZ; besides this, if the profile is validated, there is no need for scanning all possible cutting planes, which, again, enormously reduces the complexity of the traversal of the class of polyhedra targeted in this study. Finally, ZMinG and ZMinFZ also offer a way to assign a value to a profile: this value is ZMinFZ - ZMinG. If it is positive, the profile is valid. Otherwise, the more negative the value, the worse the profile.

9. RESULTS, CONCLUSION AND FUTURE WORK

The algorithms described in the previous sections have been implemented in C++ and tested on Personal Computers with an Intel Core I5-2500 processor.

Even though the optimizations presented above drastically reduce the time to traverse the whole class, it still takes months, sometimes years to do it with thin steps on a single computer when m = 7 or 8. To reduce these durations, some tests have been distributed on 60 PCs, so that an execution that would have last one year only lasts a week. Even though, thin steps (e.g. 1 degree for the angular step and 1 for the length step) remain inaccessible for m = 8. The table below gives the durations we have experienced (as if the tests had been conducted on a single computer).

m (nb of edges)	Δα and ΔL	Duration	Best profile value	Number of profiles tested
4	1° - 1	< 1 s	-18.37	102 294
5	1° - 1	3 mn	-13.95	287 331 878
6	1° - 1	30 hours	-10	307 492 054 424
7	3° - 3	4,5 days	-7.97	594 003 064 639
8	10° - 3	66 mn	-7.52	3 851 621 193
8	5° - 3	19 days	-4.66	492 902 974 042

At the moment, no CMP has been found for values of m smaller than 9. The reason may be that, because of the big steps we have to use to reduce the duration of the program when m > 6, some CMPs have escaped from the search; of course, another reason could be that there is no CMP in this class but we believe it is not the case because another experiment, that has just been launched and consists in traversing the class with an evolutionary algorithm, has found a 17 faces polyhedron that is very close from being monostatic; its value is -0.55249746 (see Figure 15).

Nearly valid half-profile	
X	Z
0	100
28.5812	83.5373
41.9472	66.1401
47.4035	49.0845
46.1614	30.7800
40.8933	17.5852
32.1911	7.12819
20.2575	0
0	0

Figure 15: the half-profile of a quasi monostatic 17-faces polyhedron

For m = 9, things are easier (and faster) as a solution is known (the half-profile of Guy's object). By scanning only the neighborhood of this profile, thousands of CMPs have been found but this is of little interest as, first, it doesn't improve Guy's record and, a fortiori, it doesn't improve Bezdek's new record.

Several extensions to this work are planned: the neighborhood of the 17 faces quasi monostatic polyhedron will be explored with the exhaustive approach and thin steps. The values of the steps will dynamically adapt during the exhaustive approach, so that the zones where the half-profiles are bad (very negative values) are scanned faster than those where the half-profiles are good (nearly valid). After that, other classes of polyhedra will be explored, beginning with a class containing Bezdek's CMP.

12.

Finally, two questions arise from the observation of Guy's and Bezdek's objects: both are built from the same profile (a sequence of 9 stuck right triangles) so : 1) is it possible to build other classes of CMPs from this profile and 2) would this profile be the common denominator of all CMPs ?

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