Animation of Water Droplets on a Hydrophobic Windshield

Nobuyuki Nakata
The University of Tokyo
5-1-5 Kashiwa-no-Ha
Kashiwa, Chiba
277-8561 Japan
nobnak@nis-lab.is.s.u-tokyo.ac.jp

Masanori Kakimoto
Tokyo University of Technology
1404-1 Katakura-machi
Hachioji, Tokyo
192-0982 Japan
kakimotoms@stf.teu.ac.jp

Tomoyuki Nishita
The University of Tokyo
5-1-5 Kashiwa-no-Ha
Kashiwa, Chiba
277-8561 Japan
nis@is.s.u-tokyo.ac.jp

ABSTRACT

Animation of water drops on a windshield is used as a special effect in advanced driving games and simulators. Existing water droplet animation methods trace the trajectories of the droplets on the glass taking into account the hydrophilic or water-attracting nature of the glass material. Meanwhile, in the automobile industry, usage of hydrophobic glass windshields has recently been a common solution for the drivers' clear vision in addition to cleaning the water with wipers. Water drops on a hydrophobic windshield behave differently from those on a hydrophilic one. This paper proposes a real-time animation method for water droplets on a windshield taking account of hydrophobicity. Our method assumes each relatively large droplet as a mass point and simulates its movement using contact angle hysteresis accounting for dynamic hydrophobicity as well as other external forces such as gravity and air resistance. All of a huge number of still, tiny droplets are treated together in a normal map applied to the windshield. We also visualize the Lotus effect, a cleaning action by the moving droplets. Based on the proposed simulation scheme, this paper demonstrates the motion of the virtual water droplets on the windshield of a running vehicle model.

Keywords

Water droplets, hydrophobicity, windshield, driving simulator, contact angle hysteresis

1. INTRODUCTION

Water flow on the window or windshield surfaces are commonly used as a rainy scene description in film works and other types of motion pictures. More recently, computer generated animations of water flow on the windshields are realized for advanced video games and driving simulators. Since the glass material has hydrophilic or water-attracting nature, water droplets move along irregular trajectories seeking for water-attracting places of the surface, as we often find on the windows in a rainy day. Most of the existing water droplet animation methods simulated these winding trajectories of the droplets.

In real driving situations, those water trajectories or water-film on the windshields due to the hydrophilicity seriously affect the visibility through the glass. To clear the water, mechanical wipers have been used since the beginning of the automobile history. In addition, as auxiliary measures, coating the windshield with water repellent material became a solution a few decades ago. In the year 2000, the first water-repellent finished windshield became commercially available. Nowadays such hydrophobic windshield products are widely used in the automobile market.

A large amount of research literature on the behaviour of water on hydrophobic surfaces is published in chemical and mechanical engineering fields. To the authors' knowledge, however, little work has been done on real-time simulation of water droplets sliding across hydrophobic windshields. In this paper, we address this problem and propose a solution consisting of several practical simulation models for use in games and driving simulators.

Water attracting or repelling feature of surface material should be quantified differently in two situations, static and dynamic. The static repellency has been investigated for a long time and the fundamentals have been established. For water droplet animation, knowledge on the dynamic repellency is more important, which is true in engineering analysis of water-shedding phenomena on the windshield. While the dynamic water repellency includes a number of unexplainable phenomena, there are a couple of major factors and indicators characterizing the dynamic repellency. Those include contact angle hysteresis, falling angle, falling velocity, and falling acceleration.

The relationship between the contact angle hysteresis and the slope angle has long been investigated. In case of an ideal water droplet shape, the contact angle hysteresis is known to be in proportion to the falling angle.

The falling velocity and acceleration vary by the surface material even when the slope angle remains constant. Although the standard methods for evaluating and measuring the falling velocity/acceleration were not established until recently, it is known that the behaviour of a falling water droplet on the hydrophobic surface is explainable in terms of rolling and sliding.

In this paper, we take the knowledge on the dynamic repellency into account and propose a real-time animation method for water droplets on the hydrophobic windshield. As the water-repellent coated windshields become standard in the automobile market, our contribution is to provide video game and simulator developers with a means of reproducing realistic and harmonious motions of the water droplet cluster traveling across the hydrophobic windshield.

This paper is organized as follows. In the next section we introduce related work on both engineering analyses and animation techniques for water droplets. Then our proposed method is explained in a theoretical point of view in Section 3, followed by more detailed descriptions on the implementation and results in Section 4. Finally we give conclusions and future work in Section 5.

2. RELATED WORK

In the computer graphics field, several methods have been introduced for animating water droplets. Kaneda et al. [Kan93a] [Kan96a] proposed methods to describe the movement of the droplets by defining each droplet as a particle and move it with particle dynamics. Since the droplets travel seeking for water-attracting places, their trajectories on the glass surface form complex shapes. They also simulated these motions by a random walk method using random numbers [Kan99a]. Recently their method was implemented as a real-time simulator with a GPU computing technique [Tat06a]. Fournier et al. [Fou98a] depicted the trajectories of droplets using the mass spring model. None of the above methods took into account the hydrophobicity of the inclined surface since they assume hydrophilicity. Also, they do not incorporate air resistance against the water drops or rolling resistance of the drops.

Several researchers have developed fluid dynamics based methods for the water droplet simulation. Wang et al. [Wan05a] took into account surface tension, contact angle, and contact angle hysteresis. The surface tension is more dominant in a water droplet than in regular large-scale fluid forms. Thürey et al. [Thu10a] introduced the mean curvature flow, which is known as a motion equation for surface boundaries, and evaluated the phenomena caused by the surface tension more appropriately than Wang et al.

Zhang et al. [Zha11a] developed a faster computation method for droplets using the mean curvature flow without other fluid simulations. They ignored the internal fluid flow of the droplets but used the surface tension and other external forces to give deformation, collision and division to each droplet represented as a polygon mesh. They achieved 10-50 fps in the experiment with 10K-50K polygon mesh. However, due to the implicit method for the mean curvature flow computation, the stability of their solution depends highly on the mesh quality and the time step, and the performance optimization is limited.

In order to tackle the problem of the droplet motion on the hydrophobic surfaces, we need to understand dynamic repellency. The structure or the behaviour of the surface molecules are considered to be a source of the dynamic repellency. To figure out the behaviour, Hirvi et al. [Hir08a] simulated a droplet consisting of thousands of water molecules using a molecular dynamics calculation technique. Korlie [Kor93a] proposed a cluster model of quasi-molecular particles on a horizontal plane and introduced its dynamical equations which lead to the value of the contact angle of the cluster.

Analyses of real water droplets have been done by several research groups. For example, Sakai et al. [Sak06a] measured the velocity and the acceleration of a droplet sliding across water-repellent surfaces. Droplets are known to run down either rolling or slipping on the incline depending on the degree of hydrophobicity [Ric99a] [Suz09a]. Hashimoto et al. [Has08a] measured the relationship between the volume and the velocity of a windswept droplet.

We address the problem of dynamic water-repellency taking the contact angle hysteresis into account. In addition, we use the knowledge of the real water drop analyses to verify and compensate our results. We avoided using the fluid dynamics simulation, the mean curvature flow, or any type of molecular forces since they are not suitable for real-time visualization. Due to the computing load and the time step limitations, those methods cannot handle sufficient number of droplets on a car windshield.

In our method, each droplet is represented as a mass point or a particle. Thus, we are able to incorporate additional forces into the real-time simulation loop; air resistance against the water droplets and viscous dissipation which acts as a rolling resistance of each drop. Although these forces are crucial factors for the fast movement of water drops, they have not been fulfilled in the previous methods [Wan05a] [Thu10a] [Zha11a].

Particle dynamics are common in the real-time simulation field. They are widely adopted in games and interactive applications. Real-time physics engines in the market are equipped with features of particle dynamics and rigid body dynamics including collision detections as fundamental functions. We implemented our method on top of a game engine 'Unity' and added unique behaviours of water droplets running slowly or quickly, or staying on the hydrophobic surfaces.

3. A PRACTICAL MODEL FOR WATER DROPLETS ON HYDROPHOBIC WINDSHIELDS

3.1 Water Droplet Geometry

When a droplet is on a solid surface, the contact angle is defined as the angle between the solid surface and the droplet surface. The contact angle is determined by the Young equation, which describes the balance of three surface tensions, as shown in Equation (1).

$$\gamma_L \cos\theta = \gamma_S - \gamma_{SL}, \quad (1)$$

where, θ is the contact angle, γ_L is the surface tension of the water droplet, γ_S is the surface tension of the solid, γ_{SL} is the boundary tension between the water and the solid (Figure 1).

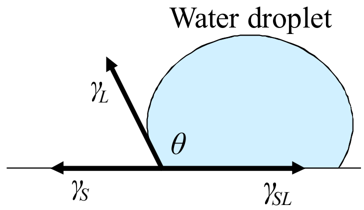

Figure 1. Contact angle and tensions of a water droplet.

When the radius of the droplet on hydrophobic surfaces is less than the radius of capillary (2.8mm), the surface tensions are the dominant factors of the water drop shape. Thus the droplet forms a near spherical geometry. Meanwhile, the contact angle of the glass becomes 90°-100° when it is coated with commercially available repellent material.

Based on the above two observations, we assume that each rain droplet is rendered as a hemisphere. In practice, the geometric shape is basically a disc-like plane and the normal vectors for refraction are controlled to make it look hemisphere. Details are described in Section 4.3.

3.2 Contact Angle Hysteresis

When a thin pipe is inserted into water, the water level in the pipe is raised by the capillary action. This is caused by a force called the capillary force which operates along the triple boundary line among the water, the solid and the air. The capillary force is determined by the Young-Laplace equation.

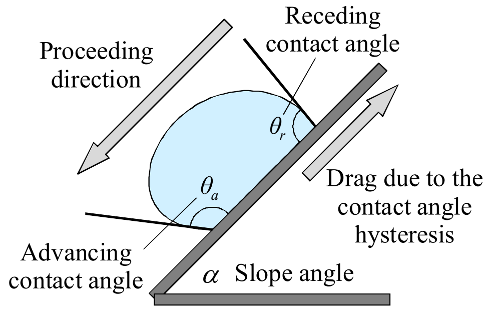

Figure 2. Advancing and receding contact angles of a water droplet.

With regard to a droplet which lies on a solid plane, the capillary forces along the circular triple boundary cancel each other out if the contact angle is constant along the circle. When some external forces are put on the droplet and its shape is deformed, the contact angles vary while the droplet stands still until the contact angle variance reaches at a certain value.

The contact angle hysteresis is defined as the difference between the advancing and receding contact angles (θ_a and θ_r, respectively). These two angles are defined as the largest and the smallest contact angles, respectively, at the moment that the water droplet starts moving on the solid plane by the sufficient external force. The slope angle at this moment is called the falling angle. Figure 2 illustrates the advancing and receding contact angles for an incline.

While the droplet is moving on the plane, a drag operates on the droplet toward the reversed direction against the proceeding direction. The amount of drag is related to the contact angle hysteresis. Assuming that the shape of the triple boundary is a circle, the drag F_{hys} is approximated with the following equation [Car95a]

$$F_{hys} \approx \frac{1}{2}\pi r \gamma_L(\cos\theta_r - \cos\theta_a), \quad (2)$$

where, r represents the radius of the water droplet. θ_r and θ_a are the receding and the advancing contact angles, respectively.

3.3 Wind Drag

Automobile windshields meet with air resistance, or wind drag, according to the velocity of the running vehicle. The wind drag is defined as follows:

$$F_{wind} = \frac{1}{2}\rho C_D S V^2, \quad (3)$$

where, ρ is the density of the air, C_D is the coefficient of resistance, S is the projected size of the droplet, and V is the velocity relative to the air.

In Equation (3), the droplet is assumed to be floating in the air. Since all droplets in our model are placed on a solid windshield, the equation needs to be modified. We assume that the wind is weakened at places very close to the solid plane. It is known that in such near-boundary layer, the wind velocity changes in a complicated manner.

We employed a simplest compensation to decrease the velocity in the near-boundary layer using an exponential law as shown in the following formula.

$$\tilde{V} = \begin{cases} V\left(\frac{y}{\delta}\right)^{\frac{1}{2}} & (y < \delta) \\ V & (y \geq \delta), \end{cases} \quad (4)$$

where, V is the wind velocity out of the boundary layer (relative to the solid plane), y is the height of the droplet, δ is a parameter representing the thickness of the boundary layer, and $\tilde{V}$ is the compensated wind velocity for the droplet.

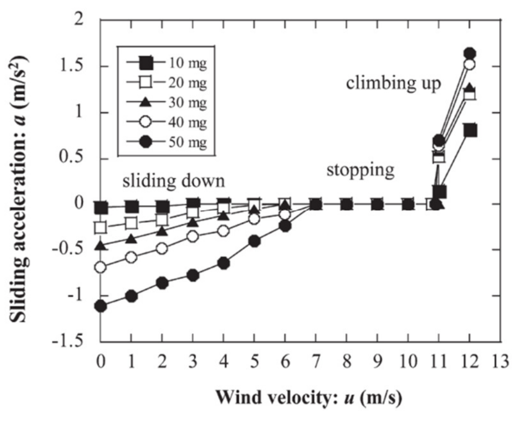

Figure 3. A measured relationship between the wind velocity and the acceleration of droplets, using a varying droplet size as a parameter (excerpt from [Has08a]).

3.4 Viscous Dissipation

When a droplet is moving or rolling, another drag is caused by some in-bulk friction called viscous dissipation [Bic05a]. The drag is in proportion to the velocity of the droplet and represented as

$$F_{bulk} = \eta R v f(\theta), \quad (5)$$

where, η is the degree of viscosity of the water, R is the radius of the droplet, v is the velocity of the droplet. $f(\theta)$ is a factor dependent on the contact angle.

3.5 Wind Speed and the Droplet Acceleration

In the surface finishing engineering discipline, Hashimoto et al. [Has08a] introduced an experiment to measure the acceleration of various volumes of water droplets placed on an angled hydrophobic plane in a wind tunnel. Figure 3 quotes from the literature and shows the result of the measured descending or ascending acceleration of the droplets. The contact angle, the slope angle, and the falling angle are 105° , 35° and 10°, respectively.

In the range where the wind velocity is relatively low, moderate but more falling accelerations are observed as the droplet size becomes greater. When the wind velocity is raised beyond a certain value (7m/s in Figure 3), the droplet stays still within some range of wind velocities. When the velocity is further raised beyond a higher value (11m/s), rapid ascending accelerations are observed, which are greater as the droplet becomes larger.

On the other hand, we simulated the sliding accelerations of a droplet taking the following five forces into account (Figure 4).

- Gravity (vertical) F_g
- Wind drag (horizontal) F_{wind}
- Perpendicular force (normal to windshield)

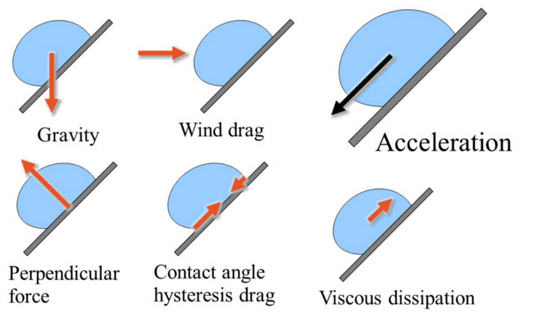

Figure 4. External forces added to a droplet and the resultant acceleration. In this example, the gravity is more dominant than the wind drag and thus the droplet slides down.

- Viscous dissipation drag (tangential to windshield) F_{bulk}
- Contact angle hysteresis drag (tangential to windshield) F_{hys}

The wind drag F_{wind} has been described in Section 3.3. The contact angle hysteresis drag behaves as a resistance force parallel to the windshield, in the same way as the perpendicular force normal to the windshield. The force F_{hys} represented in Equation (2) defines the maximum limit of the hysteresis drag.

In our implementation, the maximum limit is specified by a dimensionless coefficient $\Omega \equiv \cos\theta_r - \cos\theta_a$. Since the relationship between the wind velocity and the contact angles is hard to simulate, we approximate the Ω value as a function of the wind velocity V. When the velocity is small, we force the Ω value to keep a minimum constant Ω_{min} which is typically 0.5.

$$\Omega(V) = \max\left[\Omega_{min},\ 2\left\{1 - \exp\left(-\frac{V}{\sigma}\right)\right\}\right], \qquad (6)$$

where, σ is a constant parameter which controls the saturation rate of $\Omega(V)$. When the wind is extremely strong, the contact angles are assumed to be also as extreme as $\theta_a \to 180°$, $\theta_r \to 0°$, and thus $\Omega(\infty) \to 2$. This is well accounted for by Equation (6).

Figure 5 shows a simulated result of the accelerations for the varying droplet sizes. The range of wind velocities in which the droplet stays still is reproduced, and the range is very similar to the measured result in Figure 3.

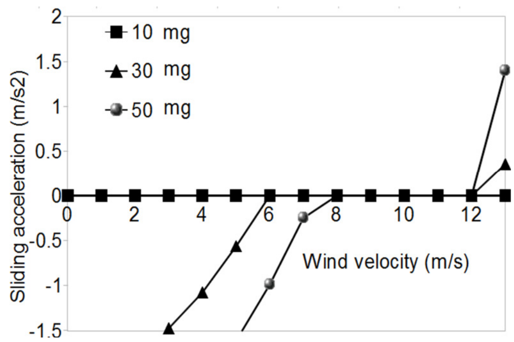

Figure 5. Simulation results of the droplet accelerations.

3.6 Collision between Droplets

The surface tension of the water droplet causes a pressure difference in the droplet. This is known as the Laplace pressure and is greater as the droplet radius is smaller. Therefore, when two water droplets of different sizes collide with each other, the small droplet gets absorbed by the larger one.

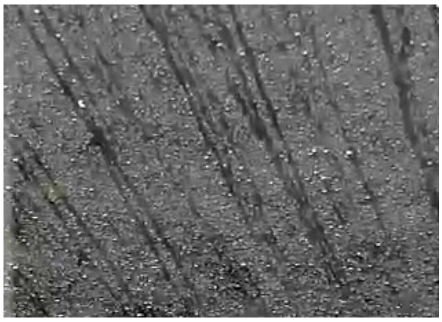

Figure 6. Droplet trajectories caused by the Lotus effect (image captured from a live-action movie of a windshield).

We implemented this process and it is invoked on droplet collision detection.

3.7 Distribution of Raindrop Radii and the Lotus Effect

Lotus effect is a phenomenon which occurs when a water droplet moves across a hydrophobic surface. Lots of very small droplets and contamination spread on the surface are removed by the moving droplet along the trajectory. The same phenomenon is observed on a windshield as demonstrated in the snapshot of Figure 6.

Figure 7, an excerpt from [Fur02a], is a rain droplet radius distribution under 1mm/h rainfall. The graph is with the raindrop diameters as the horizontal axis and the number of raindrops for each diameter as the vertical logarithmic axis. The line indicated as 'MP' is an exponential distribution model called the Marshall-Palmer distribution [Mar48]. Each graph legend is the place name of the observing site. Some legends contain observing periods in months.

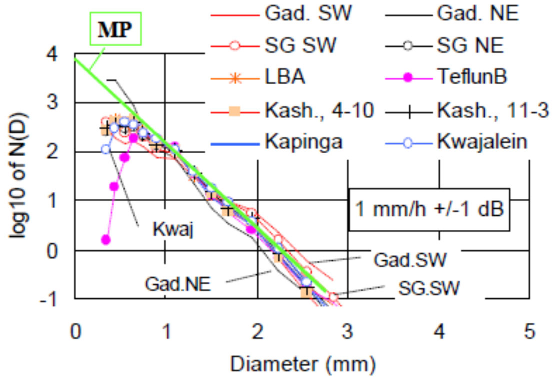

Figure 7. Distribution of the number of raindrops for each diameter (drop size distribution). Each graph legend indicates the name of the observing site (excerpt from [Fur02a]).

According to the model, the smaller the raindrop diameter is, the greater the number of raindrops becomes. Especially, tiny raindrops of below 1mm are contained with an exponentially large numbers. Therefore, it is impractical to simulate the motion of every droplet. Fortunately, those tiny raindrops do not move at all with our simulation model as shown in Figure 5. Thus we apply a single large normal map onto the windshield. The map contains the normal vectors which represents all the small droplets standing still on the windshield.

4. IMPLEMENTATION AND RESULTS

This section describes implementation of our method proposed in the previous section and demonstrates some results.

4.1 Implementation Overview

We implemented the system on top of Unity, a popular game engine. Although our method regards each water droplet as a particle, we implemented each droplet as a small rigid body which does not rotate. Regarding the rigid body physics engine, we used NVIDIA PHYSX embedded in the Unity system.

The flow of the whole process is outlined as follows.

- Initialization
- Main loop
    - Droplet generations
    - Physics simulation
    - Collision detection
        - Droplet mergers
        - Droplet deletions
    - Updates of large droplet shapes
    - Update of windshield alpha map (Lotus effect over small droplets)
    - Rendering

4.2 Physics Simulation of Droplets

In each time step of the simulation, our system calculates the external forces imposing on the water droplets as illustrated in Figure 4.

Regarding the gravity, we added some random noise to the force component parallel to the windshield in order to realize natural motions of the droplets caused by some assumed fluctuation of the running vehicle.

The implementation of viscous dissipation (Section 3.4) is a heuristic matter since the factor $f(\theta)$ in Equation (5) is not determined. We used a constant value $\eta f(\theta) = 0.5$ in the equation. The important point is that the viscous dissipation drag F_{bulk} is in proportion to the droplet velocity. The above constant value can be used to control the maximum droplet speed.

While the droplets are moved by the external forces, we obtain each collision point with its u-v coordinates and the normal vectors of the colliders from the collision detector of the physics engine. For a droplet being regarded as to be on the windshield, the windshield point corresponding to the droplet is calculated and the refraction map image for the Lotus effect is updated.

In case that a droplet collides with another droplet, the Laplace pressure effect is applied. The system compares the masses of the two droplets. If the difference is greater than the pre-defined threshold, these two will fuse together into one droplet.

4.3 Rendering Large, Movable Droplets

Each large water droplet (with over 1mm diameter) is rendered as a disc-shape polygon mesh when it is staying still on the windshield. The normal vectors on the disc surface are controlled so that the refracted environment appears to be mapped on a hemisphere.

While the droplet is moving across the windshield, its shape is deformed to be longer along the moving direction. The normal vectors are controlled so that the lengthened transparent droplet looks like a drug capsule sectioned by a screen-parallel plane. The deformation is controlled so that the assumed volume of the droplet is preserved. Using its normal vectors, the pixel shader calculates the refraction directions and maps the background texture image as the environment. Figure 8 is a close-up rendering image of a pseudo-hemisphere water droplet and a deformed pseudo-hemisphere.

Those large droplets are generated with various

Figure 8. Droplets rendered as a pseudo-hemisphere (left) and a deformed pseudo-hemisphere (right).

sizes according to the Marshall-Palmar distribution shown in Figure 7. The number of large droplets generated per frame is set to be five typically. They are accumulated but eventually moved away out of the windshield or collided and fused with others. As a result, a couple of hundred to one thousand large droplets reside in the steady-state situation.

4.4 Rendering Small and Still Droplets

Small droplets (with less than 1mm diameter) are represented as perturbation in a normal map image for the windshield, as described in Section 3.7. The diameters of the generated small droplets vary also according to the Marshall-Palmar distribution. The number of small droplets in our implementation amounts to approximately 800K$/m^3$.

The outside scene image is refracted according to the normal map. The trajectories of large droplets (pseudo-hemispheres) are stored as an image component which is used to suppress the normal map. They are composed in the shader program and the Lotus effect on the windshield surface is rendered (Figure 9).

Figure 9. The Lotus effect. Small and still droplets are rendered as a normal map on the windshield. Large and moving droplets are rendered as pseudo-hemispheres.

4.5 Performance

All results referred to in this section are captured snapshots of real-time animations rendered from the driver's point of view toward the automobile proceeding direction viewing the outside through the windshield. The source of the outside image is a motion picture shot with a video camera placed between the two front seats of a running car when no rain is falling. The pre-recorded image is mapped as a video texture onto a billboard model placed in front of the windshield model.

Figures 10 and 11 are the examples with a small wind velocity. In Figure 10, a relatively large contact angle hysteresis is specified and thus the adherence is strong that the droplets do not move at

Figure 10. A result with low wind velocity (11.3m/s) and a large contact angle hysteresis with $\Omega_{min} = 0.5$.

Figure 11: A result with low wind velocity (11.3m/s) and a small contact angle hysteresis with $\Omega_{min} = 0.05$.

Figure 12. A result with high wind velocity (15m/s) and a large contact angle hysteresis with $\Omega_{min} = 0.5$.

all. In Figure 11, the adherence is smaller and the droplets move along the windshield curve.

Figure 12 is a result with stronger wind and the large droplets climb straight up the windshield. Since the adherence is strong and the boundary layer is set to be thick, the small droplets are made still.

The frame rates for Figures 10, 11 and 12 are 134-153fps, 80-100fps, and 70-100fps, respectively. The scene contains a windshield, large droplets and the video texture billboard shapes, which total approximately 17K vertices.

4.6 Rendering Conditions

For the rendering results, we used an Intel Core2 Extreme X9600 (3GHz), NVIDIA GeForce GTX480 Graphics and 8GB main memory. The horizontal field of view was 45° and the distance between the viewpoint and the windshield was approximately 0.5m. The horizontal curvature radius of the windshield geometry was 5m constant and the vertical curvature was 0 (flat). The slope of the windshield was inclined at a 45° angle.

5. CONCLUSION AND FUTURE WORK

We proposed a real-time animation method which reproduces the behaviour of a group of water droplets on a hydrophobic windshield. We modeled each of large droplets as a mass point and took into account dynamic hydrophobicity by employing the contact angle hysteresis which causes appropriate adherence for each droplet.

We also compared the accelerations of simulated droplets with those of measured real water droplets from literature of surface finishing engineering analysis. By introducing a near boundary layer where the wind is reasonably weakened, our result matched the measured one and reproduced realistic behaviours of the droplets.

For a huge number of tiny water droplets which do not move in our model, we introduced a normal map applied to the windshield. By using the image-based droplets, the Lotus effect was effectively reproduced.

For practical number of large droplets, our method runs in real-time and can be easily adopted as an effect for video games and vehicle simulators. The performance is degraded when the large droplets are not blown off and accumulated on the windshield because the motion simulation is done on a per large droplet basis.

Future work includes the performance improvement for larger number of droplets, more realistic deformation of the droplets, and handling of uneven wind velocity distributions.

REFERENCES

[Bic05a] Bico, J., Basselievre, F., and Fermigier, M. Windswept droplets. Bulletin of the American Physical Society 2005, 58th Annual Meeting of the Division of Fluid Dynamics, 2005.

[Car95a] Carre, A., and Shanahan, M.E.R. Drop motion on an inclined plane and evaluation of hydrophobia treatments to glass. Journal of Adhesion, Vol.49, No.3-4, pp.177-185, 1995.

[Fou98a] Fournier, P., Habibi, A., Poulin, P., Simulating the flow of liquid droplets. Graphics Interface, pp.133-142, 1998.

[Fur02a] Furutsu, T., Shimomai, T., Reddy, K.K., Mori, S., Jain, A. R., Ong, J.T., Wilson, C.L. Comparison of the characteristics of the drop size distributions in the tropical zone (In Japanese). Open Workshop 2002 on Coupling Processes in the Equatorial Atmosphere, 2002.

[Has08a] Hashimoto, A., Sakai, M., SONG, J.-H., Yoshida, N., Suzuki, S., Kameshima Y., and Nakajima, A. Direct observation of water droplet motion on a hydrophobic self-assembled monolayer surface under airflow. Journal of Surface Finishing Society of Japan, Vol.59, No.12, pp.907-912, 2008.

[Hir08a] Hirvi, J.T., and Pakkanen, T.A. Nanodroplet impact and sliding on structured polymer surfaces. Surface Science. No.602, pp.1810–1818, 2008.

[Kan93a] Kaneda, K., Kagawa, T. and Yamashita, H. Animation of water droplets on a glass plate. Proc. Computer Animation ‘93, pp.177–189, 1993.

[Kan96a] Kaneda, K., Zuyama, Y., Yamashita, H. and Nishita, T. Animation of water droplet flow on curved surfaces. Proc. Pacific Graphics ’96, pp.50–65, 1996.

[Kan99a] Kaneda, K., Ikeda, S., and Yamashita, H. Animation of water droplets moving down a surface. Journal of Visualization and Computer Animation, Vol.10, No.1, pp.15-16, 1999.

[Kor97a] Korlie, M. S., Particle modeling of liquid drop formation on a solid surface in 3-D. Computers & Math. with Applications, Vol.33, No.9, pp.97-114, 1997.

[Mar48a] Marshall, J.S., and Palmar, W.McK. The distribution of raindrops with size. Journal of Meteorology, Vol.5, pp.165-166, 1948.

[Ric99a] Richard, D., and Quere, D. Viscous drops rolling on a tilted non-wettable solid. Europhys. Lett., Vol.48, No.3, pp.286-291, 1999.

[Sak06a] M. Sakai, J-H Song, N. Yoshida, S. Suzuki, Y. Kameshima and A. Nakajima, Relationship between sliding acceleration of water droplets and dynamic contact angles on hydrophobic surfaces. Surface Science, 600 (16), 204-208, 2006.

[Suz09a] Suzuki, S., Nakajima, A., Sakurada, Y., Sakai, M., Yoshida, N., Hashimoto, A., Kameshima, Y., Okada, K. Mass Dependence of rolling/slipping ratio in sliding acceleration of water droplets on a smooth fluoroalkylsilance coating. Europhys. Lett., Vol.48, No.3, pp.286-291, 2009.

[Tat06a] Tatarchuk, N. Artist-directable real-time rain rendering in city environments. Course Note #26, SIGGRAPH 2006.

[Thu10a] Thürey, N., Wojtan, C., Gross, M., and Turk, G. A multiscale approach to mesh-based surface tension flows. ACM TOG, Vol.29, No.4 (Proc. SIGGRAPH 2010), 48, 2010.

[Wan05a] Wang, H., Mucha, P.J., and Turk, G. Water drops on surfaces. ACM TOG, Vol.24, No.3 (Proc. SIGGRAPH 2005), pp.921-929, 2005.

[Zha11a] Zhang, Y., Wang, H., Wang, S., Tong Y., and Zhou, K. A deformable surface model for real-time water drop animation. IEEE TVCG, PrePrint, August 2011.