

# Nailing and Pinning: Adding Constraints to Inverse Kinematics

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## ABSTRACT

Inverse kinematics is commonly applied to compute the resulting movement of an avatar for a prescribed target pose. The motion path computed by inverse kinematics, however, often differs from the expected or desired result due to an underconstrained parameter space of the degrees-of-freedom of all joints. In such cases, it is necessary to introduce additional constraints, for instance by locking a joint's position and/or rotation. We present a method to fix a joint in terms of position and explain how to incorporate these constraints into the inverse kinematics solution.

**Keywords:** animation, inverse kinematics, positional / rotational constraints

## 1 INTRODUCTION

Many Computer Graphics applications, such as virtual environments, computer games, and interactive stories, feature animated characters, for instance humans and animals. To create animated sequences, the animator should be able to position and move all parts of the character. In many animation systems, a *skeleton* (or *articulated figure*) of the character is used to specify positions and motion. Such skeletons consist of rigid links (denoted as segments in H-Anim [HAWG] terminology) connected by joints. Usually, the articulated figure has a hierarchical structure, where each joint has its own coordinate system and is positioned relatively to the coordinate system of its parent. An articulated figure can often be divided into kinematic chains (limbs) where each chain has one end that is free to move, called the end-effector. To obtain a specific configuration of the articulated figure, each joint needs to be set to the correct rotation angle to obtain the required

position. Specifying the configuration of the figure by rotating each joint one by one down the hierarchy is denoted as *forward kinematics* (FK). In contrast, when the position and orientation of a specific end-effector is given, the rotation angles of all joints further up the hierarchy can be computed using *inverse kinematics* (IK).

We present a method to pin a joint to a position in space and explain how to incorporate these constraints into the inverse kinematics solution.

## 2 RELATED WORK

**The Resolved motion-rate method** introduced by Whitney [Whi69], is one of the methods that are frequently used to solve the IK problem. Many extensions have been proposed, such as the pseudo-inverse method [MK85], the Jacobian transpose method [Wel93] and the selectively damped least squares method [BK03]. Two approaches making use of the resolved motion-method are the weighting strategy and the task-priority approach. With the weighting strategy, such as in [BMW87], when tasks get into conflict, the algorithm will distribute the residual error among the tasks according to their weight. Therefore no task is exactly satisfied unless one task's weight is highly dominant with respect to other weights. With the task-priority approach, conflicts are dealt with directly at differential level. When all goals cannot be satisfied simultaneously, the task with the highest priority reaches its goal while the residual error of the other tasks are minimized [BB98]. We use the task-priority approach.

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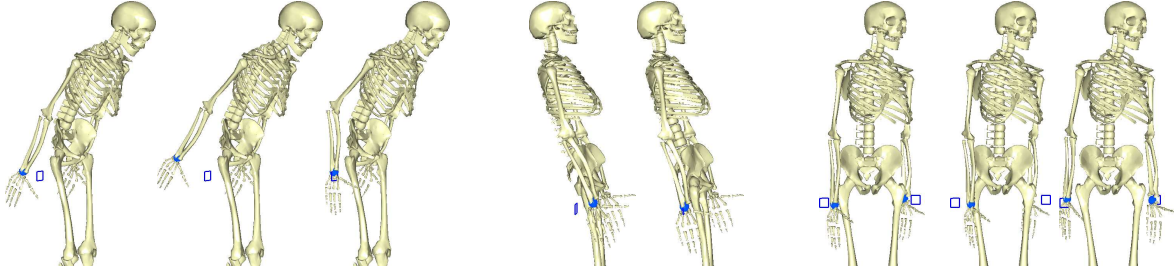


Figure 1: Constraints for keeping wrist pinned when spine is rotated. Left three images: leaning forwards moving pinned wrist; the shoulder first moves the pinned wrist away from the pinned position; the elbow moves the pinned wrist back to its pinned position. Middle two images: Moving the pinned wrist back to its position when leaning backwards. Right three images: twisting moving wrist from pinned position; the shoulder first moves the pinned wrist away from the pinned position; the shoulder moves the pinned wrist back to its pinned position

**Constraints** Rotational constraints, where the joint’s rotation about an axis is restricted within joint angle limits, are addressed [MF98] [BT97]. Badler *et al.* [BMW87] addressed positional constraints by describing how to position an articulated figure with a weighting strategy. However, joint angle limits and rotational constraints are not considered.

### 3 OUR APPROACH

#### Inverse Kinematics Method

In our system we use the task-priority algorithm with damped least-squares. We implemented the recursive algorithm, including linear equality and inequality constraints that are satisfied after each iteration step, as discussed in [BB98].

#### Dealing with Positional Constraints

To efficiently check whether a joint caused a pinned joint to move from its position, each joint  $J_i$  is assigned a chain  $K_i$  ( $i = 1, \dots, n$ ) that is used in the pinning algorithm (see Table 1). A chain  $K_i$  is a part of the hierarchy of the articulated figure that contains the joint  $J_i$ . For a hierarchy similar to the H-Anim specification [HAWG], the chain  $K_i$  usually starts at the joint  $J_i$  and proceeds down the hierarchy to the leaf nodes. For instance, the chain corresponding to the right shoulder joint would start at the shoulder, and proceed via elbow and wrist to all finger joints of the right hand. When checking whether a joint  $J_i$  caused another pinned joint to move, only joints in the chain  $K_i$  are tested. This provides the flexibility to allow a joint to be pinned for rotations by a specified chain, but not pinned for rotations by the other joints outside the chain.

The pseudo-code of our algorithm for handling pinned joints is listed in Table 1.

### 4 ADDITIONAL CONSTRAINTS

In this section, we discuss special constraints that can be used for the arm of a human-

1. for each simulation step
2.   changed = FALSE
3.   compute  $\dot{\mathbf{q}}$ ,  $\mathbf{q}$
4.    $\forall$  joints  $i = 1, \dots, n$
5.      $\theta_{\text{curr}}$  = current rotation of  $J_i$
6.      $\theta_{\text{new}} = \theta_{\text{curr}} + \dot{q}_i$
7.      $\forall$  joints  $j$  in chain  $K_i$
8.       if ( $J_j$  is pinned)
9.           $\text{pos}_{\text{curr}}$  = current position of  $J_j$
10.         set rotation of  $J_i$  to  $\theta_{\text{new}}$
11.          $\text{pos}_{\text{new}}$  = current position of  $J_j$
12.         if ( $\text{pos}_{\text{curr}} \neq \text{pos}_{\text{new}}$ )
13.            $\forall$  joints  $p = i + 1, \dots, n$
14.              $\phi_{\text{curr}}$  = current rotation of joint  $J_p$
15.              $\phi_{\text{new}} = \phi_{\text{curr}} + \dot{q}_p$
16.             set rotation of  $J_p$  to  $\phi_{\text{new}}$
17.              $\text{pos}_{\text{new}}$  = current position of  $J_j$
18.             if ( $\text{pos}_{\text{curr}} \neq \text{pos}_{\text{new}}$ )
19.                 set rotation of  $J_i$  to  $\theta_{\text{curr}}$
20.                 changed = TRUE
21.              $\forall$  joints  $t = i + 1, \dots, n$
22.                 set rotation of  $J_t$  to  $q_t$
23.             set rotation of  $J_i$  to  $\theta_{\text{curr}}$
24.             if (NOT changed)
25.                 set rotation of  $J_i$  to  $\theta_{\text{new}}$

Table 1: Our algorithm.

like articulated figure when positional constraints are added to the IK problem.

#### Reachable Space

In many applications where a reaching task is applied, it should be tested whether the goal is within the reachable space of the hand, to determine whether the spine should remain fixed or should be allowed to rotate in order to obtain a natural pose. When the shoulder position is fixed, the reachable space can be roughly approximated by a half-sphere [Zha96]. However, when there is a positional or rotational constraint set for the elbow, the method discussed in [Zha96] has to be extended to deal with these cases.

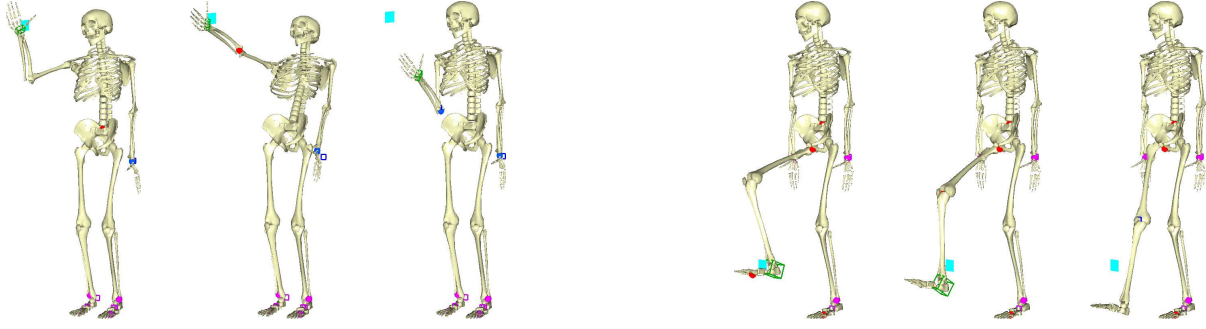


Figure 2: Different types of constraints affect the motion towards a target position. Left three images: the goal (indicated by the cyan square) is reached if no constraints are imposed on the right arm; the skeleton compensates for a rotational constraint of the right elbow by leaning backwards; the goal cannot be reached if the right elbow is pinned in space (left to right). Right three images: the right ankle reaches the goal without any constraints; a fixed rotation of the right knee prevents the ankle from fully reaching the goal; a pinned right knee makes it impossible for the ankle to get close to the goal (left to right).

If the elbow is not pinned in terms of position, the origin of the sphere is the shoulder position, the radius of the sphere is the arm length and the  $x$ -,  $y$ - and  $z$ -axis of the sphere is approximately the shoulder joint base frame. When the elbow is pinned in terms of position, the origin of the sphere is the elbow position, the radius of the sphere is the lower arm length and the  $x$ -,  $y$ - and  $z$ -axis of the sphere is approximately the elbow joint base frame.

To determine if the goal position is within the reachable space of the hand, the goal position is transformed into spherical coordinates.

Let  $\theta$  define the azimuthal angle in the  $xz$ -plane and  $\phi$  the polar angle from the  $y$ -axis. Let  $r$  be the distance from the goal position to the origin (radius). Let  $\ell_L$  define the length of the lowerarm,  $\ell_U$  the length of the upperarm and  $d$  the distance between the hand and the shoulder. Then,

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2}, \\ \theta &= \tan^{-1} \frac{x}{z}, \\ \phi &= \cos^{-1} \frac{y}{r} \end{aligned}$$

with

$$-\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}, \quad (1)$$

$$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}, \quad (2)$$

$$0 \leq r \leq \ell_U + \ell_L. \quad (3)$$

If the elbow is pinned to a position in space, Equation (3) should be changed to:

$$0 \leq r \leq \ell_L. \quad (4)$$

If the elbow has a rotational constraint, and the arm is outstretched, Equation (3) should be changed to:

$$r = \ell_U + \ell_L. \quad (5)$$

If the elbow has a rotational constraint, and the arm is not outstretched, Equation (3) should be changed to:

$$r = d. \quad (6)$$

$\theta$  and  $\phi$  should satisfy Equations (1) and (2), respectively, and  $r$  should satisfy either one of Equation (3), (4), (5) or (6), according to the constraints set for the elbow. If these equations are satisfied, the goal position is within the reachable space of the hand and the rotation of the vertebrae is fixed. Otherwise the goal position is not within the reachable space of the hand and the rotation of the vertebrae cannot be fixed.

### Spine Rotation

**Bending forwards or backwards** Assume the wrist is pinned and the spine is rotated, bending the character forward. In Figure 3 let  $\ell_L$  be the distance between the shoulder and wrist,  $\mathbf{x}$  the distance between the current and pinned wrist position and  $\mathbf{d}$  the distance between the shoulder and pinned wrist position. Angle  $B$  can then be calculated making use of the *Law of Cosines*:

$$\begin{aligned} x^2 &= \ell_L^2 + d^2 - 2\ell_L d \cos B \\ \Rightarrow B &= \cos^{-1} \left( \frac{\ell_L^2 + d^2 - x^2}{2\ell_L d} \right). \end{aligned} \quad (7)$$

The shoulder's rotation around the  $x$ -axis is set in such a way that it moves the wrist away from the pinned position. Now the rotation of the elbow that is necessary to move the wrist back to its pinned position, is calculated. In Figure 3 let  $\ell_L$  be the length of the lowerarm,  $\mathbf{x}$  the distance between the current and pinned wrist position and  $\mathbf{d}$  the distance between the elbow and pinned wrist position. Angle  $B$  can now be calculated as discussed above. When the spine rotates, bending the character backwards, only the rotation for the shoulder should be calculated. This is illustrated in Figure 1.

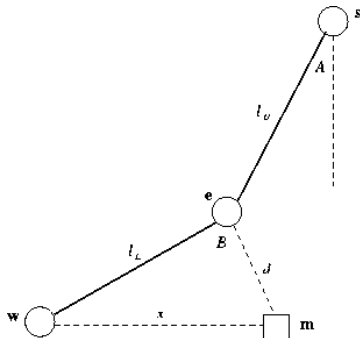


Figure 3: Triangle that is formed between the shoulder, elbow, and wrist.

**Twisting** Assume the wrist is pinned and the spine is rotated, resulting in a twist. Then, in Figure 3 let  $\ell_L$  be the distance between the shoulder and wrist,  $\mathbf{x}$  the distance between the current and pinned wrist position and  $\mathbf{d}$  the distance between the shoulder and pinned wrist position. The rotation of the shoulder is calculated as discussed above. The rotation of the shoulder around the  $y$ -axis is set such that the wrist is moved towards its pinned position. Then the rotation of the shoulder around the  $x$ -axis is calculated in the same way, moving the wrist towards its pinned position. This is illustrated in Figure 1.

## 5 RESULTS

**Rotating spine** When the spine is rotated and the wrist is pinned, the rotation will move the wrist from its pinned position, as can be seen from the first pose from each group in Figure 1. Then by adding the constraints discussed in Section 4, the wrist is moved back to its pinned position.

**Reachable space** On the left of Figure 2 the task is to move the right wrist upwards towards the goal position. When there are no constraints on the arm, the goal position is within the reachable space (discussed in Section 4) of the hand and therefore the vertebrae do not rotate. When the elbow is fixed, the goal position is not within the reachable space of the hand (the distance to the goal position is not equal to the length of the arm) and therefore the rotation of the vertebrae are not fixed. To compensate for the rotational constraint of the elbow, the character leans backwards to reach the goal. When the elbow is pinned to a position in space, the goal position is further away than the length of the lowerarm and therefore not within the reachable space of the hand. In an attempt to get as close as possible to the goal position, the skeleton turns towards the goal, but the goal cannot be reached.

The full version of the paper with a detailed discussion, as well as a comparison

between our method and the weighting strategy, can be found at: <http://www.mpi-sb.mpg.de/resources/VirtualHumans/publ/wscg2005.pdf>

## 6 FUTURE WORK

In future we want to apply the algorithm to motions such as walking and jumping, e.g. walking up or down stairs, where one foot needs to stay at a position while stepping downwards or upwards. Another possible application could be a ballet dancer that needs to keep the hand on the bar while doing the ballet movements and where in many movements one foot has to stay pinned at a certain position. We also want to extend the algorithm to include the possibility to pin a joint to a relative position in space, e.g. that the hands stay on the back of the dancing partner while they are dancing, i.e. moving in space.

## REFERENCES

- [BB98] P. Baerlocher and R. Boulic. Task-priority formulations for the kinematics control of highly redundant articulated structures. In *IEEE IROS '98*, pages 323–329, 1998.
- [BK03] S.R. Buss and J-S. Kim. Inverse kinematics with selectively damped least squares, July 2003.
- [BMW87] N.I. Badler, K.H. Manoochehri, and G. Walter. Articulated figure positioning by multiple constraints. *IEEE Computer Graphics & Applications*, 7(6):28–38, June 1987.
- [BT97] R. Boulic and D. Thalmann. Interactive identification of the center of mass reachable space for an articulated manipulator. In *Proceedings of International Conference of Advanced Robotics (ICAR)*, pages 589–594, July 1997.
- [HAWG] Web3D Consortium (H-ANIM) Humanoid Animation Working Group. H-Anim: Specification for a standard VRML Humanoid, version 1.1. <http://www.h-anim.org/Specifications/H-Anim1.1>.
- [MF98] N. Madhavapeddy and S. Ferguson. Specialised constraints for an inverse kinematics animation system applied to articulated figures. In *Proc. Eurographics UK '98*, 1998.
- [MK85] A.A. Maciejewski and C.A. Klein. Obstacle avoidance for kinematically redundant manipulators in dynamically varying environments. *International Journal of Robotics Research*, 4(3):109–117, 1985.
- [Wel93] C. Welman. Inverse kinematics and geometric constraints for articulated figure manipulation. Master's thesis, Simon Fraser University, 1993.
- [Whi69] D.E. Whitney. Resolved motion rate control of manipulators and human prostheses. *IEEE Transactions on Man-Machine Systems*, 10(2):47–53, 1969.
- [Zha96] X. Zhao. Kinematic control of human postures for task simulation. Technical Report IRCS Report 96–32, University of Pennsylvania, December 1996.