Modeling with Rational Bézier Solids

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ABSTRACT
In this paper we present a technique for modeling solids based on the rational trivariate Bézier expressions. These solids are defined by analytical expression. For modeling purposes we focus on rotational, transitional and twisted solids. Final visualization is then done by approximation of solids by net of points and by boundary evaluation of solid. We also present practical output of our visualization algorithm.

Keywords
rational Bézier solids, rotational, transitional, twisted, solids visualization

1. INTRODUCTION
The trivariate solids are not very popular in geometric modeling because of high degree of freedom. Rather they were used for free-form deformations and modeling [Sed86]. But, for some part of geometric modeling, they are useful. It is very easy to map 3D textures on such solids, also some properties can be attached to the points of solid with trivariate functions. Some types of solids can be used for deformation simulation because of their transformation invariance. Also, there exist works considering approximations of these solids by subdivision schemes [Mcd02]. In this paper we focus on one group of trivariate solids, rational Bézier solids.

2. RATIONAL BÉZIER SOLIDS
The rational Bézier tetrahedral is defined with a degree n, tetrahedral domain $ABCD$, control net of points, and for each point one real number (weight). Control net with weights can be written in following way:

$$V_{\mathbf{i}} \in E^3; w_{\mathbf{i}} \in R$$
$$\mathbf{i} = (i, j, k, l); |\mathbf{i}| = i + j + k + l = n; i.j, k, l \geq 0$$

Let us have point U from the domain and let $\mathbf{u}=(u,v,w,t);u+v+w+t=1$ are barycentric coordinates of point U ($U=uA+vB+wC+tD$) with respect to $ABCD$. Point of rational Bézier tetrahedra $RB^n(\mathbf{u})$ can be defined using analytical expression:

$$RB^n(\mathbf{u}) = \frac{\sum_{|\mathbf{i}|=n} w_{\mathbf{i}} V_{\mathbf{i}} B_{\mathbf{i}}^n(\mathbf{u})}{\sum_{|\mathbf{i}|=n} w_{\mathbf{i}} B_{\mathbf{i}}^n(\mathbf{u})}; B_{\mathbf{i}}^n(\mathbf{u}) = \frac{n!}{i!\,j!\,k!\,l!} u^i v^j w^k t^l$$

Rational Bézier tensor solid is defined with three degrees n, m, o, box domain $ABCDEFGH$ and a control net of points and for each point a real number (weight):

$$V_{\mathbf{i}} \in E^3; w_{\mathbf{i}} \in R$$
$$\mathbf{i} = (i, j, k); n \geq i \geq 0; m \geq j \geq 0; o \geq k \geq 0$$

Assume that we have point U from domain and let $\mathbf{u}$ are coordinates of U with respect to $ABCDEFGH$, so $\mathbf{u}=(u,v,w);0\leq u,v,w\leq 1$. Now we can define point of Bézier tensor solid $RB^{n,m,o}(\mathbf{u})$ with analytical expression:

$$RB^{n,m,o}(\mathbf{u}) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{o} w_{(i,j,k)} V_{(i,j,k)} B_i^n(u) B_j^m(v) B_k^o(w)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{o} w_{(i,j,k)} B_i^n(u) B_j^m(v) B_k^o(w)}$$

where

$$B_i^n(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

are Bernstein polynomials.

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3. MODELING

Translational solids

Let us have a rational Bézier patch with given control points $P_{(i,j)}$ and weights $s_{(i,j)}$, where $i=0,..,n;j=0,\ldots,m$ and rational Bézier curve with control points Q_i and weights r_i, where $i=0,\ldots,o$. The curve and patch must correspond to each other, they share their first vertices, so $P_{(0,0)}=Q_0$. Then translational rational Bézier tensor solid has degrees *(n,m,o)* and following control points and weights:

$$V_{(i,j,k)} = P_{(i,j)} + Q_k - Q_0; w_{(i,j,k)} = s_{(i,j)} * r_k$$
$$0 \le i \le n; \qquad 0 \le j \le m; \qquad 0 \le k \le o$$

Rotational solids

Lets have rational Bézier curve with control points $Q_i=[Qx_i, Qy_i, Qz_i]$ and weights r_i, where $i=0,\ldots,o$. Then rotational rational Bézier tensor solid has degrees *(2,2,o)* and the following control points and weights:

$$V_{(0,0,i)} = [Qx_i, Qy_i, Qz_i]; w_{(0,0,i)} = r_i;$$
$$V_{(0,1,i)} = [Qx_i + Qy_i, Qy_i - Qx_i, Qz_i]; w_{(0,1,i)} = r_i;$$
$$V_{(1,0,i)} = [Qx_i - Qy_i, Qx_i + Qy_i, Qz_i]; w_{(1,0,i)} = r_i;$$
$$V_{(0,2,i)} = [Qy_i, -Qx_i, Qz_i]; w_{(0,2,i)} = 2r_i;$$
$$V_{(1,1,i)} = [0, 0, Qz_i]; w_{(1,1,i)} = r_i;$$
$$V_{(2,0,i)} = [-Qy_i, Qx_i, Qz_i]; w_{(2,0,i)} = 2r_i;$$
$$V_{(1,2,i)} = [Qy_i - Qx_i, -Qx_i - Qy_i, Qz_i]; w_{(1,2,i)} = 2r_i;$$
$$V_{(2,1,i)} = [-Qx_i - Qy_i, Qx_i - Qy_i, Qz_i]; w_{(2,1,i)} = 2r_i;$$
$$V_{(2,2,i)} = [-Qx_i, -Qy_i, Qz_i]; w_{(2,2,i)} = 4r_i;$$

where $i=0,\ldots,o$.

Twisted solids

The twisted solid is like a translational solid, but when we are translating patch along the curve, on each level we rotate translated control points around given axis by the given angle. So assume that we have rational Bézier patch given by control points $P_{(i,j)}$ and weights $s_{(i,j)}$, where $i=0,..,n;j=0,\ldots,m$ and a rational Bézier curve with control points Q_i and weights r_i, where $i=0,\ldots,o$. We will be rotating each level patch around z-axis by angle α. The control points and weights of resulting rational Bézier tensor patch with degrees *(n,m,o)* are:

$$V_{(i,j,k)} = (P_{(i,j)} + Q_k - Q_0) * \begin{pmatrix} \cos(k\alpha) & \sin(k\alpha) & 0 \\ -\sin(k\alpha) & \cos(k\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$w_{(i,j,k)} = s_{(i,j)} * r_k$$
$$0 \le i \le n; \qquad 0 \le j \le m; \qquad 0 \le k \le o$$

4. RESULTS & CONCLUSION

Based on rational Bézier solids and its approximation by point nets, we visualized some solids modeled using described ways. We have presented trivariate solids based on Bézier notion for splines and prepared basic modeling tools for creating such solids

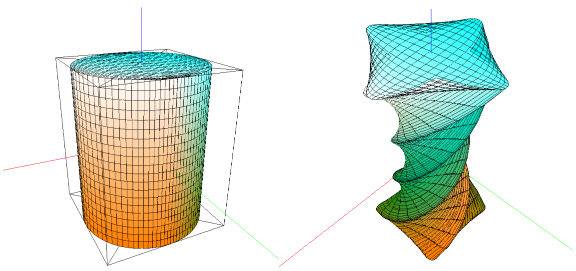

Figure 1. Transitional and twisted rational Bézier tensor solids.

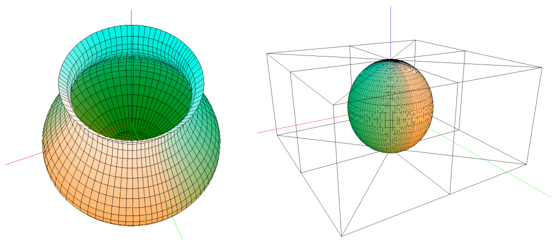

Figure 2. Rotational rational Bézier tensor solids

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