Tree Growth Visualization

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ABSTRACT

In computer graphics, models describing the fractal branching structure of trees typically exploit the modularity of tree structures. The models are based on local production rules, which are applied iteratively and simultaneously to create a complex branching system. The objective is to generate three-dimensional scenes of often many realistic-looking and non-identical trees. Our goal, instead, is to visualize the growth of a prototypical tree of certain species. It is supposed to look realistic but, more importantly, has to conform with real, measured data. We construct a tree model being similar to existing ones and extend it by coupling the branching production rules with dynamic tree-growth rules. The latter are based on equations derived from measured street tree data for London Plane tree (Platanus acerifolia) such as tree height, diameter-at-breast-height, crown height, crown diameter, and leaf area. We map the global, measured parameters to the local parameters used in the tree model. The mapping couples knowledge from plant biology and arboriculture, as we deal with trees that are trained and manipulated to achieve desired forms and functions within highly urbanized environments.

Keywords
Tree Growth, Animation, L-systems, Scientific Visualization.

1 Introduction

Several methods exist in computer graphics to describe and model computer-generated trees. Their common goal is to generate, from scratch, photorealistic images of many trees of a selected species. The trees are designed to appear as natural as possible, one species at a time. Many trees of one species should vary in appearance, so that together they resemble a naturally grown forest stand.

Few approaches have tackled the animation of trees growing over time, as the growing process of a plant is complex and many biological phenomena need to be considered. To verify and validate tree models, authors typically refer to the human eye, which is easy to fool. The generated images are supposed to appear "natural", i. e., as if they were exact copies of trees as they occur in natural settings. To our knowledge, none of these approaches verify their tree models by quantitatively comparing tree dimensions and other parameters with measured data from actual trees.

We generate tree growth animations from tree dimensions measured by scientists with the USDA Forest Service, Center for Urban Forest Research at the University of California, Davis. Their street tree growth study considered parameters such as tree height, crown height, crown diameter, diameter-at-breast-height, and leaf area. The study was conducted in Modesto, California and led to equations for predicting these parameters and their correlation. Tree growth equations were used with numerical models to estimate the annual benefits for pollutant uptake, energy savings, rainfall interception, carbon dioxide sequestration, and property value increase. We describe the study and measured parameters in Section 3.

When modeling a tree with a computer system, the structure of the tree is usually described procedu-

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rally, where the model comprises information about the branching system such as branch length, branching angle and twist, or fractal dimension. Typically, the modularity of tree branching structures is exploited by defining local production rules, that are applied iteratively to create complex branching systems. The tree model parameters are local and the production rules are applied to each branch individually, while the measured parameters from the study are global, describing the overall shape and appearance of the tree. In Section 4, we describe the tree model we have chosen for our purposes; and in Section 5, we describe how the globally measured parameters are mapped to parameters locally controlling tree growth (using the tree model) and how the tree is grown in an animation.

2 Related Work

Computer graphics has been using formal descriptions for the modeling, simulation, and rendering of trees and plants for decades. The formal description is typically based on local production rules. Starting from the trunk of a tree, the production rules generate new branches and are applied iteratively to the individual parts of the tree until the desired branching structure is reached. This method of generating plants is based on the assumption of plant modularity, which leads to repeated patterns being observed throughout the plant structure. The production rules are typically "context-free" or "context-sensitive" in the context of formal languages.

The most common representative of such formal descriptions for tree modeling is the so-called *L-system* or *Lindenmayer-system*, named after the theoretical biologist Aristid Lindenmayer who introduced the concept in [Lin68]. Prusinkiewicz and Lindenmayer developed many algorithms to model different species with different characteristics, all based on L-systems [PL90]. One major development was the introduction of parametric L-systems, where the production rules depend on the values of some locally stored and updated parameters. Other authors picked up on their methods and developed them further. Examples can be found in [AK85, Blo85, LD99]. In [FH79], a tree model is discussed that maximizes total leaf area while varying the branching geometry. A survey of existing L-system approaches is given in [PHMH95].

In [PHHM97], more emphasis is given to how the L-system model applies to nature. Life, death and reproduction are discussed, as well as information flow in growing plants. Also, the influence of the environment on the growth of plants is considered.

For computer graphics applications such as computer-animated movies or video games, the main objective of modeling plants is to generate a scene being highly realistic. Stochastic tree models have been introduced to simulate variety within one species. The individual plants can be organized in a sophisticated way to create forests or fields [CSHD03] and even entire ecosystems [DCSD02].

The methods mentioned above are mainly targeted toward the generation of static tree models. To animate plant development, L-systems can be extended to *dL-systems* or *differential L-systems*, as introduced in [PHM93]. The production rules of dL-systems are parametric, where the values of the parameters are defined as the solution of differential equations. Recently, implicit surface representations for growing trees were used in [GMW04]. Inverse modeling techniques were used to define the tree structure and its development.

3 Tree Parameters

The study of street tree species underlying our method was conducted by the Center for Urban Forest Research. Tree size, management, and site conditions were measured for twelve common street tree species in the San Joaquin Valley city of Modesto, California. However, for the tree growth visualization described in this paper, we focus on one species, namely, the London Plane tree (Platanus x acerifolia). The 27 randomly sampled London Plane trees were planted from two to 89 years ago. The study is described in detail in [PMM01].

Data collected for each tree during June through September 1998 include species, age, address, diameter-at-breast-height, tree height, crown diameter in two directions (maximum and minimum axis), height to the base of crown, and leaf area. Observational data include a visual estimate of crown shape, pruning level, tree condition code, and planting location (front lawn, planting strip, or sidewalk cutout).

Condition code was calculated as per the Guide for Plant Appraisal (Council of Tree and Landscape Appraisers 1992). Pruning level estimation, distinguishing between no pruning, less than 10% of crown pruned, 10% to 39% pruned, and 40% or more pruned, was based on total percentage of crown removed due to crown raising, reduction, thinning, and heading during the last four-year pruning cycle. As trees matured, pruning included crown raising. Mature tree maintenance typically consisted of crown cleaning and thinning.

Two digital photos of each tree crown, taken at perpendicular angles (chosen to provide an unobstructed view of the crown) were used to estimate leaf area using an image processing method [PM98, PM03]. Ages of trees for which age data were missing or entered incorrectly in the database, were verified through searching handwritten planting records, interviewing residents and city arborists, or increment coring to count growth rings. Crown height was calculated by subtracting the bole height (distance to base of crown) from total tree height.

Typically, street tree databases include diameter-at-breast-height size classes but rarely any age information for each tree. Therefore, in this study only diameter-at-breast-height was regressed on age; all other variables were regressed on diameter-at-breast-height (DBH), enabling users to predict the other dimensions using measures of diameter-at-breast-height alone. Three curve-fitting models were tested to a small sample of healthy trees. A logarithmic regression model provided the best fit for predicting all parameters except leaf area, for which a non-linear exponential model was used. The resulting functions for DBH, height, crown diameter, crown height, and leaf area of the London Plane trees are shown in Figure 1.

In the following, we refer to these functions as $f_{DBH}(t)$, $f_H(t)$, $f_{CD}(t)$, $f_{CH}(t)$, and $f_{LA}(t)$, respectively, where parameter t is time. Visual observation of the data revealed increasing variability with age and size of the trees. Therefore, we assumed the error to be multiplicative as is indicated by the confidence intervals shown in the graphs. A complete description of the analysis and models, including the necessary standard error of estimates, response sample mean and correlation values needed for calculating confidence intervals are available on the Center for Urban Forest Research website[1].

4 Tree Model

The canonical parts of a branching structure are bifurcations and branches. In plant science, they are referred to as nodes and internodes, respectively. Due to the modularity of nodes and internodes, repeating patterns, and the fractal structure of trees, computer models typically use iteratively, simultaneously, and locally applied production rules to generate complex branching structures. Thus, the entire tree can be generated based on local operations and local parameters.

A branch or internode is defined by its length l, diameter d, start point $\mathbf{s}$, and direction $\mathbf{l}$, as shown in Figure 2(a). A bifurcation or node is defined by the angles ϕ_i between the axes of the parent branch and the child branches and by the ratios in length $\frac{l_i}{l_0}$ and diameter $\frac{d_i}{d_0}$ between the parent branch and the child branches, $i = 1,2$, as shown in Figure 2(b). When one of the child branches bifurcates again, it will, in general, not lie in the same plane but in a plane of different orientation. The change in orientation is defined by the divergence or twisting angle θ_i, $i = 1,2$. In addition to the nodes and internodes, there are leaves and flowers. No production rules are applied to leaves and flowers, but they can grow in size s. In our application, we only require leaves, but for other applications flowers can be integrated in the same way.

We use a parametric L-system to describe our tree model. The chosen parametric L-system can be defined as a context-free or context-sensitive grammar $G = (V,T,S,\Pi)$, where the set of variables V consists of branches $B(l,d,\mathbf{s},\mathbf{l})$ and the trunk $T(l,d,\mathbf{s},\mathbf{l})$, the set of terminals T consists of leaves $L(s)$, the start symbol

[1] `http://wcufre.ucdavis.edu`

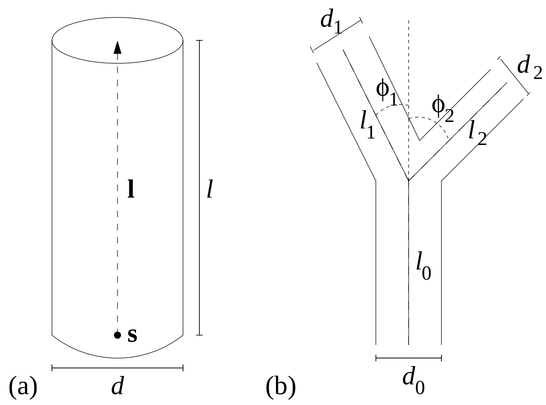

Figure 2: Branching structure consists of internodes (a) and nodes (b).

is the trunk $T(l,d,\mathbf{s},\mathbf{l})$, and a set of production rules, which are defined in the remainder of the paper. The trunk is, in principle, also a branch, but its parameter values cannot be derived from a parent branch, as there is none, which requires us to treat the trunk separately.

The branching structure is stored in a binary tree. In nature, there may occur, for example, ternary branching, but we are applying our methods to urban street trees that are frequently pruned. Since ternary branching is not beneficial for robust and balanced tree growth, such structures are regularly removed during pruning.

Each branch in the binary tree stores length l and diameter d. Start point $\mathbf{s}$ and direction $\mathbf{l}$ are not stored in a global coordinate system, but are computed in a local coordinate system with respect to the parent branch. Thus, each branch stores an orientation in form of a bifurcation angle ϕ and a divergence angle θ. To control growth of branches over time, we also store its time of creation t_0 and some growth factors, whose use is explained in the subsequent section. Growth can be limited by storing maximum length and diameter, which are, again, computed from the parameters of the parent branch.

5 Tree Growth

To grow a tree, we have to extend the static L-system tree model by introducing a continuous time dimension. Prusinkiewicz et al. [PHM93] enhanced parametric L-systems by solving differential equations to update local parameters. This so-called dL-system treats the solving of differential equations in the same way as the application of update rules. Thus, depending on the values of the considered parameters either production rules are applied or differential equations are solved for these parameters.

Since we are using measured values and since our goal is to visualize the measured data, there is no need to define plausible differential equations. Instead, we can directly incorporate the functions derived in Section 3

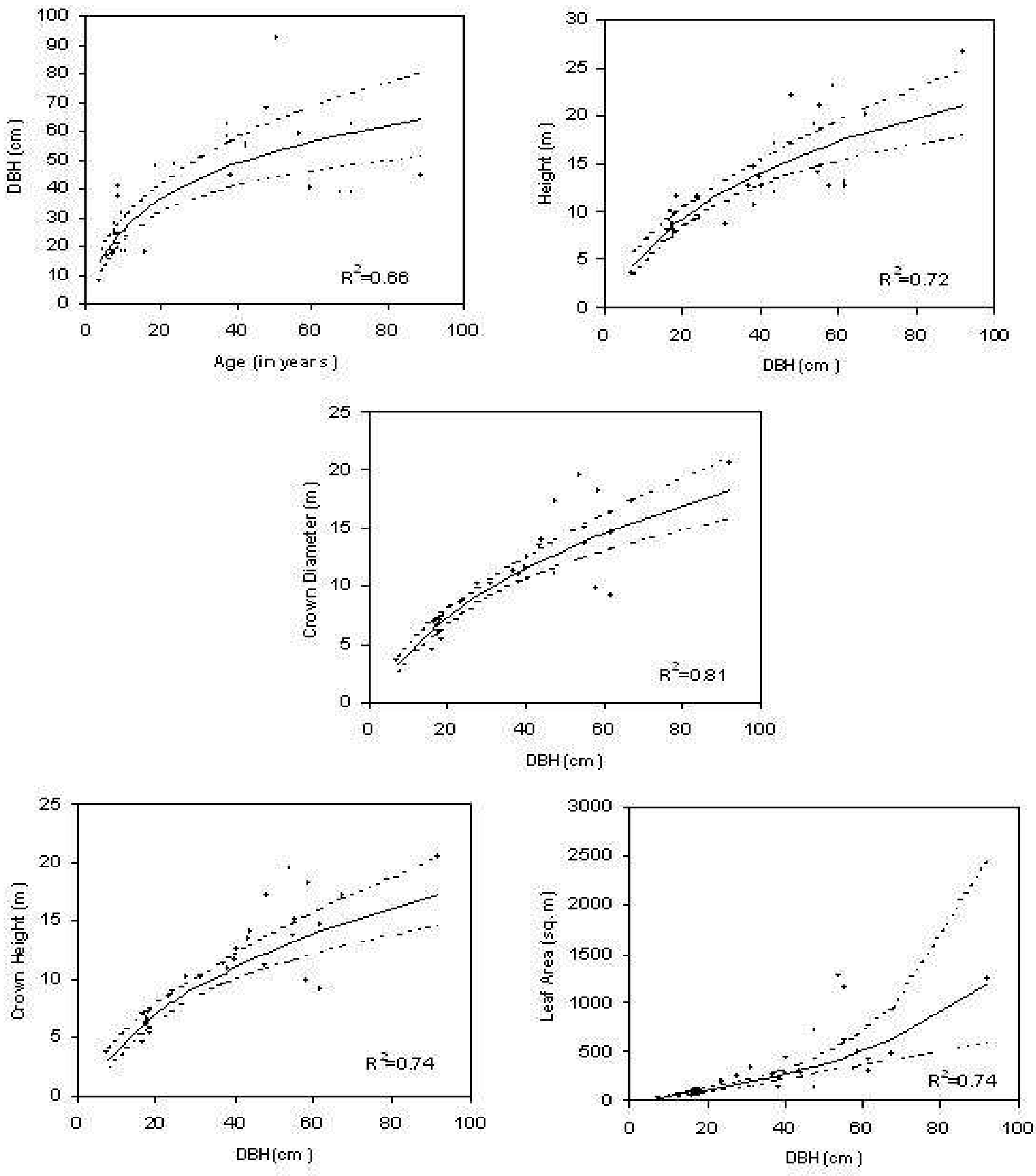

Figure 1: Experimental data for diameter-at-breast-height (DBH), height, crown diameter, crown height, and leaf area of London Plane trees.

and shown in Figure 1. The functions describe how the measured parameters are supposed to be updated over time. It remains to be explained how these measured global parameters are used to update and control the local parameters of the dynamic L-system. We make use of certain facts known from plant biology, see [HKVF88, KK79, Nik94]. We describe the relevant parameters for our model next.

Trunk length and diameter.
The length l and the diameter d of the trunk are directly controlled by the global functions. The length $l = l(t)$ is defined as the difference between the measured height of the tree and the measured height of the crown, i. e.,

$$l(t) = f_H(t) - f_{CH}(t) \ .$$

The diameter $d = d(t)$ is directly proportional to the measured DBH, i. e.,

$$d(t) = c_{DBH} \cdot f_{DBH}(t) \ ,$$

where $c_{DBH} \in [1, 1+\varepsilon)$ for a small $\varepsilon > 0$.

Branch length.
When a branch grows, it exhibits a similar growth rate as the trunk or the tree as a whole. Thus, the length of a branch follows the growth rates of the respective functions. To assure that our tree model has the actual, measured tree height, we use the function $f_H(t)$ to control the elongation of internodes.

Intuitively, primary branches (i. e., branches that emanate from the main branch/trunk) start growing before secondary branches (i. e., branches that emanate from primary branches) exist, and so on. Thus, primary and secondary branches do not grow at the same rate; while primary branches may already have reached a slow-growing phase, the secondary branches may still be in their initial fast-growing phase (Figure 1). This fact requires us to keep track of the time of creation t_0 of a branch and to compute the growth with respect to this point in time.

Moreover, a secondary branch does not reach the length of a primary branch, and a tertiary branch does not reach the length of a secondary branch, etc. Therefore, we multiply the growth function with a scaling coefficient c_l. The scaling coefficient c_l of a branch is obtained from the scaling coefficient of its parent branch multiplied by the scaling factor $s_l \in (0,1)$, where the trunk has a scaling coefficient c_l of value one. The scaling factor s_l depends on the species. For the London Plane tree, we use random values $s_l \in (0.6, 1)$. The randomness is required to make the tree appear less symmetric and thus more realistic.

In summary, the length $l = l(t)$ of a branch at time t is given by

$$l(t) = \frac{l_{max}}{H_{max}} \cdot c_l \cdot f_H(t - t_0) \ ,$$

where l_{max} and H_{max} are the maximum length of the branch and the maximum measured height of the tree, respectively. The maximum length l_{max} of a branch is determined by the maximum length of the parent branch multiplied by the scaling factor s_l. The growth of the branch terminates when the maximum length is reached.

Branch diameter.
When a branch bifurcates, the child branches have a smaller diameter than the parent branch. Leonardo da Vinci postulated that the square of the parent's diameter is the sum of the squares of the diameters of the children. In a dynamic setting, we use the measured function $f_{DBH}(t)$ multiplied by a scaling coefficient c_d to determine the growth of the diameter $d = d(t)$.

The scaling coefficient c_d is based on the scaling coefficient c'_d of the parent branch but also on the scaling coefficient c_l, which establishes a correlation between the scaling in length and diameter. The scaling coefficient is computed as $c_d = c_l \cdot c'_d \cdot (1 - 0.7 \cdot c'_d)$. The trunk has a scaling coefficient c_d of value one. Different diameters for different branches are induced by the randomness in the scaling coefficient c_l.

The growth in diameter is computed with respect to the time of creation t_0. The diameter $d = d(t)$ is defined by

$$d(t) = c_d \cdot f_{DBH}(t - t_0) \ .$$

Branch orientation.
The orientation of a branch is determined by the orientation of the parent branch, the bifurcation angle ϕ, and the divergence angle θ. The bifurcation angle ϕ is based on the ratio of crown diameter $f_{CD}(t)$ and crown height $f_{CH}(t)$, which defines the shape of the crown. London Plane trees are vertically ellipsoidal, which means that their crown height is greater than crown diameter. The ratio of crown diameter $f_{CD}(t)$ and crown height $f_{CH}(t)$ is approximately constant over time. We define the bifurcation angle by

$$\phi = \arctan\left(\frac{f_{CD}}{f_{CH}}\right) \pm \alpha \ ,$$

where α is a small random angle to make the tree less symmetric and thus more realistic.

When choosing divergence angles θ, we have to consider that we are visualizing urban street treesregularly pruned to obtain an “optimal” shape. A balanced tree, where primary branches called scaffolds are evenly spaced radially around the trunk, is considered optimal. Also, lower branches are removed to allow for clearance by trucks. Therefore, we choose

$$\theta = 130^\circ \ ,$$

which results in evenly spaced branches spiraling up the trunk.

It remains to discuss how to decide when to apply the update rules leading to tree growth and when to apply production rules leading to bifurcation. Our approach is to grow each branch using the update rules, until the branch has reached its maximum length, and to create a new branch using the production rules, once the branch has reached its maximum length.

Leaves are grown on all branches being smaller than a predefined threshold. The threshold is, again, dependent on the species. Leaves spiral around the branch at a set interval and have randomized orientation.

6 Results and Discussion

To visualize tree structure, we render each branch as a cylinder. The stored diameter d is always the diameter at the beginning of a branch. The diameter at the end of the branch is determined by the diameter of the adjacent branch. For an ending branch, which has no child branches, the cylinder degenerates to a cone.

We have taken digital photographs of both the bark and the leaves of a London Plane tree, which we use as textures for the branches and leaves in our renderings. We have modified the bark texture such that the texture can be wrapped around a branch without discontinuities in the transition area and such that multiple copies of the textures can be stitched together without discontinuities.

We animate tree growth by using the tree model of Section 4 and the local growth parameters discussed in Section 5, which are used to visualize the global parameters from Section 3. Snapshots of the animation, taken at ages $t = 10$, 20, 30, 40, and 50 years, are shown in Figure 3.

To obtain a better feeling for the dimensions of the tree, we add context in form of a human standing next to the tree. For reference, we also display the age of the tree during animation.

In addition to the quantitative, measured parameters that directly influence the visual appearance of the tree, we are also interested in visualizing quantitative benefit-cost parameters. The dollar (US) value of annual benefits for the London Plane tree in Modesto were numerically modeled for energy savings, air pollutant uptake, CO_2 sequestration, stormwater runoff reduction, and aesthetics [PM03]. Average annual costs for the same species were based on an analysis of tree work records for plant/water, prune, remove, infrastructure repair, and storm clean-up. Their values are displayed by benefit-cost bars animated to reflect the typical stream of benefits and costs over time for this species in Modesto.

The results (Figure 3) are quite satisfactory, as we successfully animate growth of a realistic-looking London Plane tree over 50 years, while conforming to measured tree dimensions. The emphasis of our work was not to make the tree look as realistic as possible but to display its growth in terms of trunk height and width, crown height and width, and leaf area. Growth of these parameters is represented in a visually appealing and intuitive way. For example, one can observe how the diameter of the trunk increases steadily but with a decreasing rate due to the fact that the trunk grows a new ring every year, but annual ring width decreases over time.

Although we do not have a video recording available of a real tree growing over time, we can, at least, compare visually the results in Figure 3 with the digital photograph shown in Figure 4. Our goal was not to replicate this particular tree in Figure 4, but to grow a prototypical London Plane tree.

We use knowledge from plant biology where possible, e. g., to estimate certain coefficients needed to map measured parameters to our tree model parameters. On the other hand, we have developed methods for urban street trees, where pruning practices modify tree architecture. Thus, certain concepts, such as natural death of certain branches or leaves or information flow in growing plants as described in [PHHM97], are not relevant. Instead, we complement biological knowledge with arboricultural knowledge, for instance, to estimate the orientation and spacing of scaffold branches.

The appearance of our tree could still be improved. The growth direction of the branches would benefit from more equal spacing [FH79]. The concept of having branches grow toward the sky and toward least-crowded areas could be introduced. This concept also includes the thinning of branches in the tree's interior caused by lack of light. The implementation of this concept would require us to change the tree model, as it requires global information; our tree model only stores local information. For example, when growing one branch, we can only retrieve information about the branch itself and its parent and child branches. We cannot retrieve information about spatially close branches, which, if known, would allow us to bend the current branch to achieve an equal distribution.

Figure 4: Digital photograph of a London Plane tree in the San Joaquin Valley city of Modesto, California.

7 Conclusions and Future Work

We have introduced an approach to model and visualize the growth of urban street trees. Growth is controlled by measured, global parameters such as tree height and width, crown height and width, and leaf area. We map these measured parameters to the local parameters of a computer-generated tree model. The tree model is based on a formal description using locally, iteratively, and simultaneously applied production rules, which exploit the modular structure of trees and allow for easy modeling of fractal branching. The production rules are coupled with local update rules that describe the dynamic growth of individual branches. The update rules are based on functions derived from measured data. Hence, we can animate and visualize the growth of a realistic-looking tree based on real data. The animation also includes additional benefit-cost parameters.

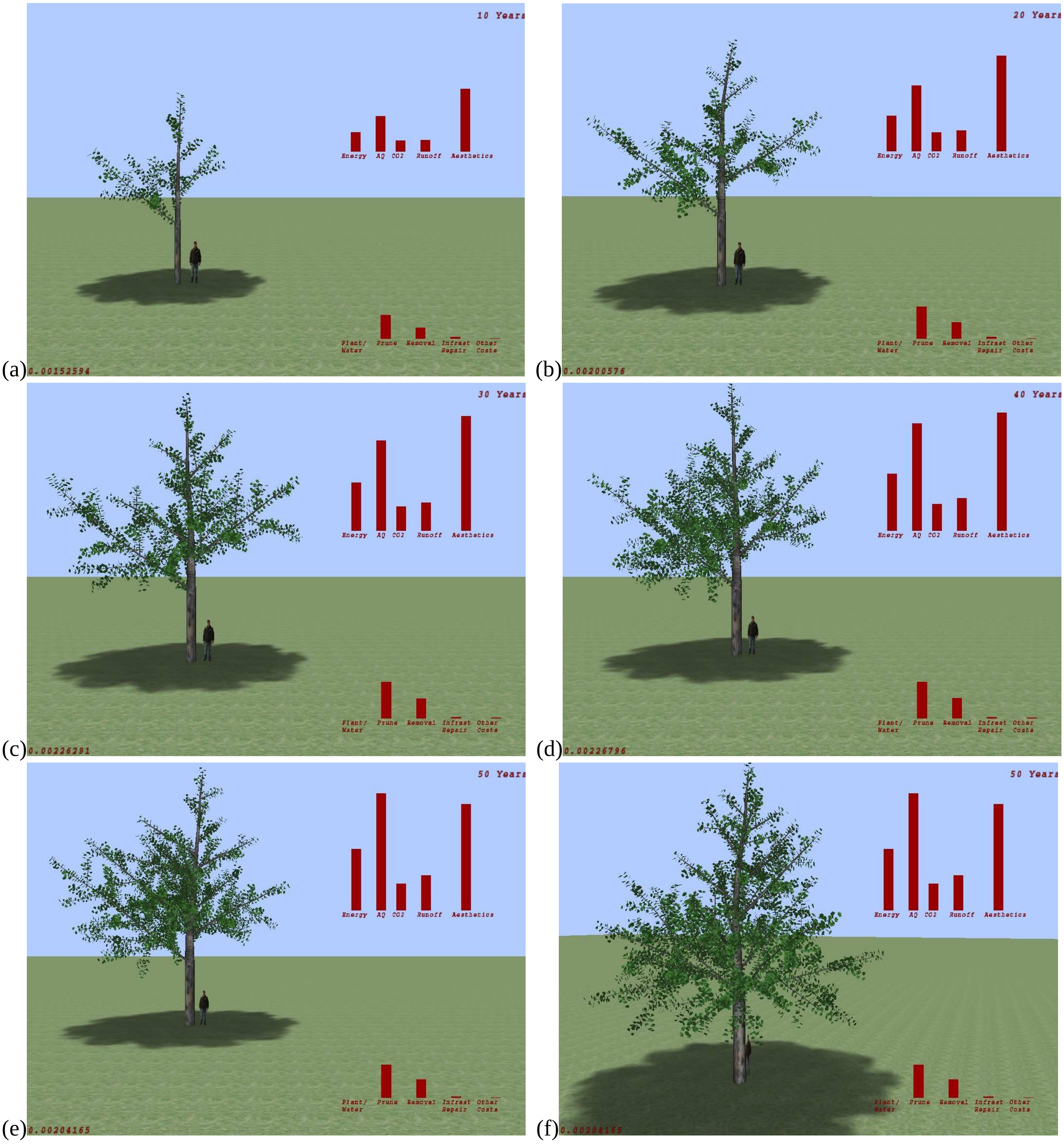

Figure 3: Tree growth visualization of a London Plane tree; ages shown: (a) 10 years, (b) 20 years, (c) 30 years, (d) 40 years, and (e),(f) 50 years.

We have used our method for modeling the London Plane tree, whose parameters were measured in the San Joaquin Valley city of Modesto, California. Up to now, we have used only this species, but we plan to use our methods for all twelve street tree species of the San Joaquin Valley study. This information will help gardeners, designers, planners, and tree managers to decide which species are most appropriate to grow in terms of size, form, benefits, and costs. Because trees are long-term investments, selecting the right species is critical to achieving maximum net benefit. The application of our methods to other species is straight forward, as it only requires us to exchange the growth functions derived from the measured values, and to use the appropriate textures. In addition, the species-dependent coefficients c_{DBH}, s_l (controlling c_l and c_d), and l_{max} and the threshold for growing leaves must be adjusted.

We plan to enhance our tree model by adding capability to store and retrieve global shape and structure information of the tree. The local structure, which is based on production rules, makes it easy to model the fractal branching structure of a tree, but limits the control of global shape.

By coupling the L-system-based model with a data structure capable of retrieving global shape information, we hope to achieve a more equal branch distribution, where branches grow in preferred directions. Global shape control will make it easier to match crown shape more precisely.

Retrieving global information from the tree model also will facilitate fast computation of leaf area at any time during animation. Thus, leaf area could be compared to measured data and the derived leaf-area growth function $f_{LA}(t)$. We plan to use leaf area to control the branching time of individual branches, the number and distribution of leaves, and the fractal dimension of the branching structure.

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References

[AK85] M. Aono and T.L. Kunii. Botanical tree image generation. *IEEE Computer Graphics & Applications*, 4(5):10–34, 1985.

[Blo85] J. Bloomenthal. Modeling the mighty maple. In B. A. Barsky, editor, *Computer Graphics (SIGGRAPH '85 Proceedings)*, volume 19, pages 305–311. ACM SIGGRAPH, 1985.

[CSHD03] M. Cohen, J. Shade, S. Hiller, and O. Deussen. Wang tiles for texture and image generation. In A.P. Rockwood, editor, *SIGGRAPH '03 Proceedings*. ACM SIGGRAPH, 2003.

[DCSD02] O. Deussen, C. Colditz, M. Stamminger, and G. Drettakis. Interactive visualization of complex plant ecosystems. In M. Gross, K.I. Joy, and R.J. Moorhead, editors, *Proceedings of IEEE Visualization '02 conference*. IEEE Computer Society Press, 2002.

[FH79] J. Fisher and H. Honda. Branch geometry and effective leaf area: a study of terminalia-branching pattern; 1: Theoretical trees, and pattern; 2: Survey of real trees. *American Journal of Botany*, 66:633–655, 1979.

[GMW04] Callum Galbraith, Lars Muendermann, and Brian Wyvill. Implicit visualization and inverse modeling of growing trees. *Computer Graphics Forum (Proceedings of Eurographics 2004)*, 23(3), 2004.

[HKVF88] H.T. Hartmann, A.M. Kofranek, V.E.Rubatzky, and W.J. Flocker. *Plant Science: Growth, Development and Utilization of Cultivated Plants*. Englewood Cliffs, NJ, Prentice-Hall, 1988.

[KK79] P.J. Kramer and T.T. Kozlowski. *Physiology of Woody Plants*. Academic Press, New York, 1979.

[LD99] B. Lintermann and O. Deussen. Interactive modeling of plants. *IEEE Computer Graphics & Applications*, 19(1), 1999.

[Lin68] A. Lindenmayer. Mathematical models for cellular interaction in development. *Journal of Theoretical Biology*, 18:280–315, 1968.

[Nik94] K.J. Niklas. *Plant Allometry: The Scaling of Form and Process*. University of Chicago Press, Chicago, 1994.

[PHHM97] P. Prusinkiewicz, M. Hammel, J. Hanan, and R. Mech. Visual models of plant development. In G. Rozenberg and A. Salomaa, editors, *Handbook of Formal Languages, Vol. III: Beyond Words*, pages 535–597. Springer-Verlag, Berlin, 1997.

[PHM93] P. Prusinkiewicz, M. Hammel, and E. Mjolsness. Animation of plant development. In J.T. Kajiya, editor, *Computer Graphics (SIGGRAPH '93 Proceedings)*, volume 27, pages 351–360. ACM SIGGRAPH, 1993.

[PHMH95] P. Prusinkiewicz, M. Hammel, R. Mech, and J. Hanan. The artificial life of plants: Artificial life for graphics, animation, and virtual reality. In *SIGGRAPH '95 Course Notes*, pages 1–38. ACM SIGGRAPH, 1995.

[PL90] P. Prusinkiewicz and A. Lindenmayer. *The algorithmic beauty of plants*. Springer-Verlag, New York, 1990.

[PM98] P.J. Peper and E.G. McPherson. Comparison of five methods for estimating leaf area index of open-grown deciduous trees. *Journal of Arboriculture*, 24(2):98–111, 1998.

[PM03] P.J. Peper and E.G. McPherson. Evaluation of four methods for estimating leaf area of isolated trees. *Urban Forestry & Urban Greening*, 2:19–29, 2003.

[PMM01] P.J. Peper, E.G. McPherson, and S.M. Mori. Equations for predicting diameter, height, crown width, and leaf area of San Joaquin Valley street trees. *Journal of Arboriculture*, 27(6), 2001.