# Skeletonization of Two-Dimensional regions Using Hybrid Method


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## ABSTRACT

Conversion of two-dimensional objects into a skeletal representation forms an essential step in many image processing and pattern recognition applications. Most of the topological structure of objects, and the information contained in the outline of their shapes, are preserved in the skeleton. Approaches based on Voronoi techniques preserve topology, but heuristic measures are introduced to remove unwanted edges. Methods based on Euclidean distance functions can localize skeletal points accurately, but often at the cost of altering the topology of the object. In this paper we offer a method to generate skeletal representations combining these two methods, which is robust and accurate, and preserves topology.



### Keywords

Shape description, Medial axis, Topology preservation, Voronoi diagram, Distance transform.


## 1. INTRODUCTION

Shape representation and description plays an important role in most computer vision systems. A useful and reliable shape representation must meet a number of requirements, which include invariance, uniqueness, and stability [Mok92]. If two objects have the same shape, then their representations should be the same and should be invariant with respect to translation, rotation, and scaling. Uniqueness means that if two objects have different shapes they should have different representations. Stability denotes the fact that if two objects have a small shape difference, then their representations should have a small difference. Conversely, if two representations have a small difference, then the objects they represent should also have a small shape difference. Therefore, a stable representation means a representation that is insensitive to noise.



The representation should reflect the shape of an object at various levels of abstraction and should also combine both boundary and region information of the object. Finally, the shape descriptors and the recognition of objects should be efficiently computable.

The skeleton of a two-dimensional object is a transformation of the shape object into a one-dimensional line. Skeleton representation as introduced by Blum [Blu67] meets most of these requirements.

Since the introduction of the skeleton shape descriptors, many skeletonization algorithms have been reported in the literature [Smi87] [Lee93]. Existing skeletonization approaches can be classified into two categories: discrete methods (thinning methods, grassfire methods, potential field methods and map distance).continuous methods (using Voronoi diagram).

Map distance methods implement the idea of medial axis transformation in straightforward way [Ros66] [Nil97], but there seem to be serious problems finding a correct and connected set of discrete skeleton elements due to several problems when dealing with discrete metrics and balls. Potential field methods [Kég02] [Sid99] [Tek98] avoid some of these problems by tracking field lines and potential valleys in continuous space. Also the potential function is much smoother than the minimal boundary distance, and so potential fields are less sensitive to the noise. However, they require much computational effort and number of smartly chosen seed points to guarantee a complete and centered skeleton.

Voronoi methods have the advantage of being very well defined and theoretically sound since they operate on well known and powerful concept, the Voronoi diagram. However, there are two critical points: the transition from discrete to continuous space (the diagram structure depends heavily on the boundary sampling) and the robust pruning of spurious branches. One advantage of Voronoi methods is the fact that the object's initial connectedness is directly transferred to the diagram, whereas other methods have to restore connectedness artificially in a post-processing step [Att97] [Fab02]. On summary, discrete methods can localize skeletal points accurately, but often at the cost of altering the object's topology and being noise sensitive. Continuous methods (using Voronoi diagram), preserve topology, but heuristic post-processing are introduced to remove unwanted edges to preserve the homotopy, but then they are less sensitive to the noise.

A mixed skeletonising method is introduced in this paper. Our method is based on the combination of two techniques in order to regroup the advantages of each one, such as homotopy preservation, good localisation, robustness to the noise. After a brief description of the previous work, the distance map method and Voronoi skeleton are exposed in section 2 and 3 respectively, our method is presented and discussed in section 4. Finally, concluding comments are made in section 5.

# 2. SKELETON DETECTION FROM DISTANCE MAP

The first approach to skeletonization appeared in the year 1967, when Blum introduced the concept of a skeleton in his paper about the so-called medial axis transform [Blu67]. It is based on distance maps which have been used subsequently for different purposes.

## Calculating the Distance Map

Computing a distance map $(DM)$ is not a very difficult task in discrete space. Given a suitable metric, one can compute the distance transform by propagating distance values from the boundary inwards [Ros66] [Nil97] [Kég02][Bor86].

The DM labels each pixel with the distance to the closest background pixel on the object. An integer distance value is used as an approximation to the true Euclidean distance for efficiency. The DM calculation uses a weighted distance metric: a pixel's direct neighbours are a distance $d_D$ away, and its indirect neighbours a distance $d_I$ away (Fig 1), where $d_D < d_I$. It has been shown in [Thi94] that if $d_D$=1, and $d_I$=1.351 will produce the best approximation to the Euclidean distance, it is not robust with regard object rotation , we also show that the pseudo- Euclidean metric is not the true distance , but it is the more robust measure will be under object rotation. We have used $d_{34}$ metrics ($d_D$=3, $d_I$=4) for our study (Fig 2). Other integer distance metrics approximate more closely the Euclidean measure but include a larger neighbourhood (5*5)[Bor93].

| | | |
|---|---|---|
| 4 | 3 | 2 |
| 5 | *p* | 1 |
| 6 | 7 | 8 |

**Figure 1: Neighbor labeling**

**Direct Neighbors** $N_D = \{1,3,5,7\}$

**Indirect Neighbors** $N_I = \{2,4,6,8\}$

| | | |
|---|---|---|
| 4 | 3 | 4 |
| 3 | *p* | 3 |
| 4 | 3 | 4 |

**Figure 2: $d_{34}$ mask**

We have implemented a two-pass algorithm to calculate the DM [Ros66]. The distance value of each pixel in the object is initialised to some large integer value, and exterior pixels are initialised to zero. For each pixel $p_i$ the distance value $d(p_i)$ is calculated by taking the distance value of each of the pixel's neighbours and adding $d_D$ if the neighbour is directly adjacent or $d_I$ if it is a diagonal neighbours (Fig 3). The pixel's new distance is set to the minimum of these calculated distances. This operation is efficient because each pixel stores pointers to its eight nearest neighbours. Two passes are made: the first processes from top to down and left to right, the second processes from bottom to top and right to left. In each pass, the minimum distance to the boundary is updated for each pixel.

Pass 1:

$$d(p_i) = \min\left(d(N_2) + d_D, d(N_3) + d_I, d(N_4) + d_D, d(N_5) + d_I\right)$$

Pass 2:

$$d(p_i) = \min\left(d(p_i)\ , d(N_6) + d_D\ , d(N_7) + d_I\ , d(N_0) + d_D\ , d(N_1) + d_I\right)$$

| | | |
|---|---|---|
| 4 | 3 | 4 |
| 3 | 0 | |

**(a)**

| | | |
|---|---|---|
| | 0 | 3 |
| 4 | 3 | 4 |

**(b)**

**Figure 3: Sequential mask $d_{34}$.**

**(a) From top to down, (b) from bottom to top**

## Identifying Local Maxima in the Distance Map

The set of axial points is derived from the Distance Map by applying the Medial Axes Transform (MAT). If the Distance Map is represented as a 3D surface, with the height corresponding to the distance values, the MAT is the set of local maxima $M$ . Many skeletonization approaches [San96] [Kim95] [Mal98], originating with the work of Blum, utilize a distance Map/MAT approach, but with varying the local maxima detection. In our work we used a natural approach for detecting the skeleton: compute the principals curvature of the distance map and detect skeleton locations where these principals curvature (gaussian or mean curvature) are not high (Fig 4).

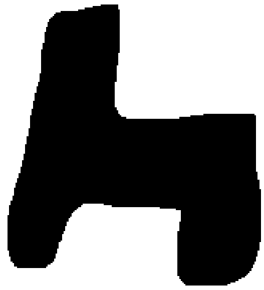
Gaussian curvature:

$$K = \frac{\rho_{xx}\rho_{yy} - \rho_{xy}^2}{\left(1 + \rho_x^2 + \rho_y^2\right)^2} \qquad (1)$$

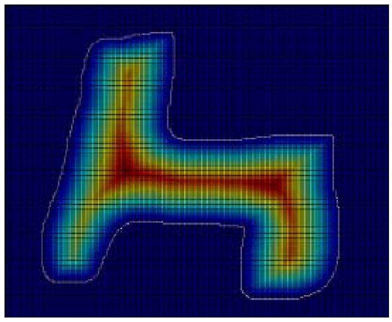
Mean curvature:

$$2H = \frac{\left(1+\rho_x^2\right)\rho_{yy} - \rho_x\rho_y\rho_{xy} + \left(1+\rho_y^2\right)\rho_{xx}}{\left(1+\rho_x^2+\rho_y^2\right)^{3/2}} \qquad (2)$$

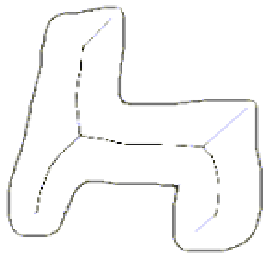
where $\rho_x$ is the first order derivative according to *x*, $\rho_y$ the first order derivative according to *y*, $\rho_{xx}$ the second order derivative according to *x*, $\rho_{yy}$ the second order derivative according to *y*, $\rho_{xy}$ the second order derivative according to *x* and *y*.



(a)



(b)



(c)

**Figure 4: Skeleton by distance map**

**(a) Object**

**(b) Distances map**

**(c) Skeleton by mean curvature**

## 3. VORONOI DIAGRAM SKELETON

### Voronoi Diagram

Among the algorithms known for computing Voronoi diagrams of points in 2D, 3D and higher dimensions are the divide-and-conquer algorithm proposed by Shamos [Pre90] and Fortune's sweepline algorithm [For87].

Given a set of points $\Omega = \{p_i\}$ so-called sites in a plan $E$, the convex polygonal region $V(i)$ containing only site $p_i$, the Voronoi polygon of $p_i$, is defined as the set of all points lying closer to $p_i$ than to any other site $p_j$:

$$V(i) = \{p, p \in E | \forall j, j \neq i : d_2(p, p_i) \leq d_2(p, p_j)\} \quad (3)$$

where $d_2$ denotes the distance between two points for the conventional Euclidean $L_2 - metric$

The collection of boundaries $\partial V(p_I)$ of all $V(p_i)$ is called Voronoi diagram $(VD)$ or Voronoi tessellation of $\Omega$, $Vor(\Omega)$

$$Vor(\Omega) = \bigcup_{p_i \in \Omega} \partial V(p_i) \quad (4)$$

## Skeleton from Voronoi Diagram

Let us assume that the planar shape $S$ is sufficiently approximated by polygon $B(S)$, whose vertexes $\overline{B(S)} = \{p_i\}$ have been obtained by equidistant sampling of $\partial S$ with sampling density $\varepsilon$. Schmitt [Sch89] show that the centers of a subset of Delaunay balls converge towards the medial axis when the sampling density of the boundary object increases. More precisely, Shmitt proved that when $\varepsilon$ tends to 0, the centers of all the Delaunay circles converge towards the medial axis.

The 2D Voronoi skeleton of an object can be computed from set points from the objects boundary. The skeleton is obtained from the dual of the Delaunay triangulation of the sample point, a skeleton is a subgraph of the Voronoi tessellation of the point.

As stated in [Att95] [Att97], the 2D Voronoi skeleton has been defined in many ways : Voronoi vertexes included in $S$ Fig 5.a, Voronoi elements included in $S$ Fig 5.b, intersection of Voronoi diagrams with $S$ Fig 5.c. However, it seems most convenient to define the skeleton as the Voronoi vertexes and edges included in the object, since this is the smallest set which approximates the medial axis.

Boundary sampling is a crucial point for Voronoi skeletonization of discrete objects.

On one hand, exact matching of the original shape would require an infinite set of sample points (and thus, sites). On the other hand, each additionally introduced site will increase the number of vertexes and edges in the resulting Voronoi diagram. Similar problems arise as an effect of boundary noise. Each spurious or additional boundary point immediately increases the complexity of the Voronoi diagram.

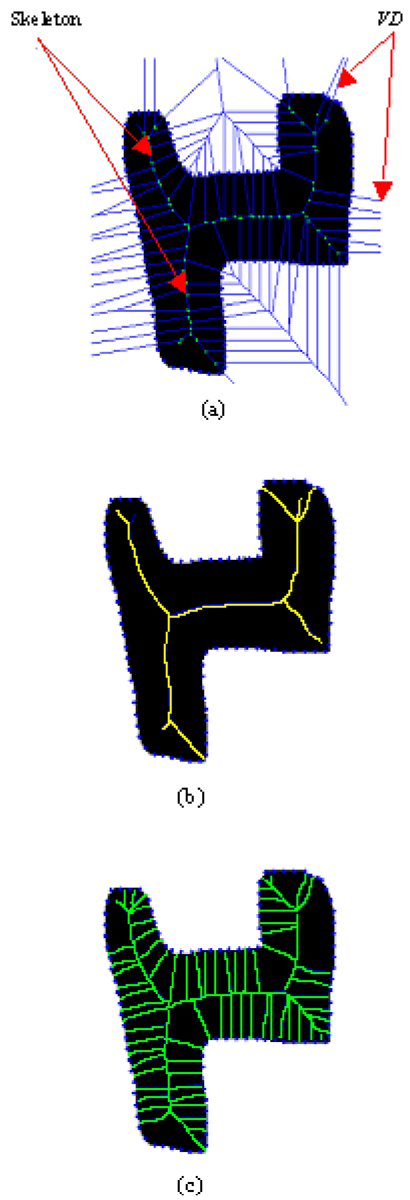



**Figure 5: Voronoi diagram skeleton**

**(a) Voronoi vertexes included in** $S$

**(b) Voronoi elements included in** $S$

**(c) Intersection of Voronoi diagrams with** $S$

## 4. MIXED METHOD

In the process of skeletonising an object, some conditions must be respected such as the homotopy, the reversibility and the correct localization of the skeletal. Furthermore, one aims to obtain the graph structure without the need of a post-processing procedure. In the present work, we propose a method that is able to satisfy these conditions. This method is obtained by the combination of two previous ones: the first is based on the map distance, and the second uses Voronoi diagram. The approaches based on Voronoi techniques preserve homotopy and appear largely invariant with respect to geometric transformations like rotation and translation. Unfortunately, they introduce heuristic measures in order to remove unwanted edges. For this reason we propose to localize the skeletal segments by the use of medial axis detection based on distance map.

Our approach may then be summarized. First by the application of Voronoi method to the objects contour in order to preserve homotopy. Then, in a second steps the segment candidates are removed by computation of local maxima in the distance map, such that the better fitted ones will remain.

## Algorithm

The following processing steps were used to obtain the final skeleton.

1. A conventional Voronoi diagram according to [Att95] is used to extract the complete Voronoi diagram of the object *(VDobj)*.

2. A distance map skeleton *(DMS)* is computed according method presented in section 2.

3. The automatic pruning procedure is used to mark all edges *VDobj* which are members of the Distance Map skeleton.

In order to prune the `VDobj` to its stable inner branches, we have proposed a simple algorithm. For any vertex of Voronoi segments if a disc of radius 3 centred at this vertex, intersects the Distance Map skeleton we preserve this segment, otherwise we remove it (Fig 6).

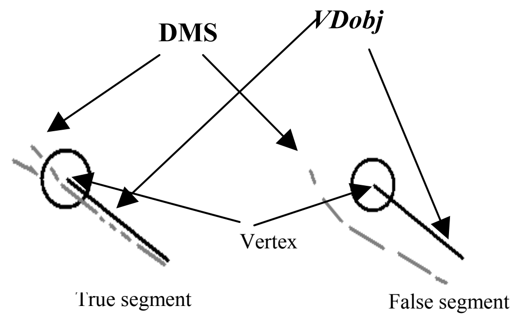



**Figure 6: Pruning segments**

## Robustness

Figure 7 depicts the skeletons extracted by the above automatic pruning process. This figure illustrates the robustness of the proposed automatic pruning when confronted with a series of shapes, the degree of boundary distortion increasing from top to bottom.

Even if there are significantly jagged boundaries, the algorithm correctly identifies a salient subset of the skeleton which shows large structural resemblance to the medial axis Transform on an ideal rectangle. Satisfying results are also obtained if holes are added to the shape (Fig 8.c). Analysis of our skeletons promises to assist a more flexible definition of shape similarity.

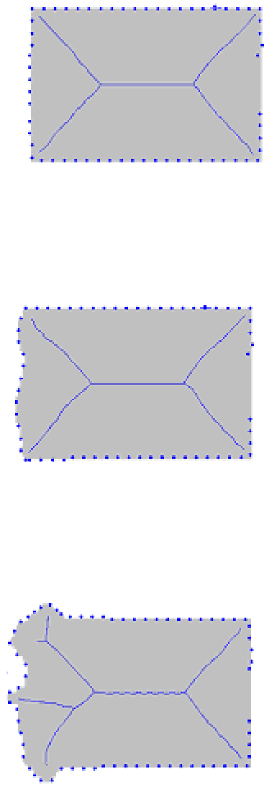


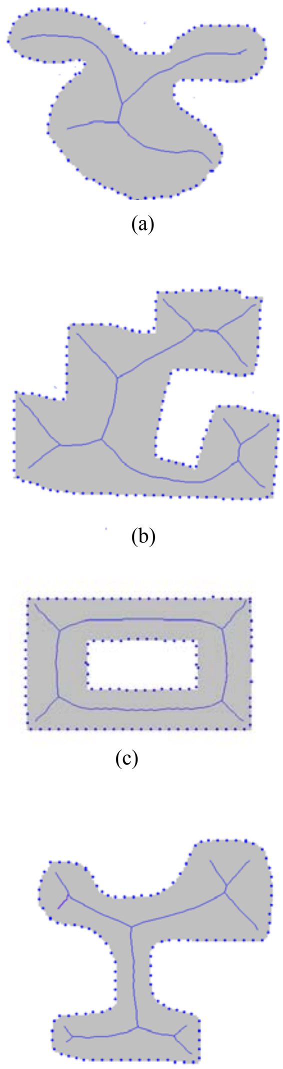
**Figure 7: Robustness test**

## Homotopy

The main motivation to develop our algorithm is the preservation of the homotopy (the object and the skeleton are topologically equivalent). This one is preserved naturally by the structure of the Voronoi's graph that constructs a continuous skeleton. This is reinforced by the fact that the intersection of this graph with the distance Map skeleton forces the resulting skeleton to stay inside the shape.

To verify that our algorithm preserves the homotopy of the object, we have done several tests.

In figure 8, we notice that in spite of a weak sampling, the structure of the skeleton is the same of the object.



(a) (b) (c) (d)

**Figure 8: Homotopy test**

## 5. CONCLUSION

The goal of this work was to explore possibilities of how to extract a skeleton from two dimensional images.

In order to overcome some of the limitations of previous distance map based approaches and Voronoi Skeleton, an alternative method has been proposed which tries to exploit both the distance map information and the connectivity information contained in Voronoi diagram of the sampling boundary object's. By the intersection of the two skeletons, the complete connectivity, the homotopy and a geometric invariance can be guaranteed any time, but then the localisation is not better due to the sampling contour.

The implemented skeletinization algorithm has been tested on various images, and experimental results have been presented and discussed. The algorithm is able to extract a connected set of lines which is a desired skeleton. The major drawback at this stage of the work is the fact that the resulting graph depend on the boundary sampling.

Our future work is to find automatically a better sample of the objects contour that don't modifies the skeleton appearance. A second important direction of future research is to generalize this algorithm for 3D objects in order to extract a 3D curve skeleton.

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