Improving Form Factor Accuracy by Projecting onto the Hemisphere Base

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ABSTRACT

In this work we introduce a new method for computing Form Factors in Radiosity. We demostrate how our method improves on existing projective techniques such as the hemicube. We use the Nusselt analog to directly compute form factors by projecting the scene onto the unit circle. We compare our method with other form factor computation methods. The results show an improvement in the quality/speed ratio using our technique.

Key words

Global Illumination, radiosity, form factor, Z-buffer, simple plane, polar plane, projection techniques.

1. INTRODUCTION

Radiosity is one of the most important techniques for the synthesis of realistic images with global illumination. One of the keys of the application of the radiosity method is the computation of the form factors. The form factor of a patch *i* to another patch *j* specifies the fraction of total energy emitted from *i* that arrives at patch *j*.

The formula of the calculation of the form factor of a patch *i* to another *j* is :

$$F_{i,j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} V_{i,j} dA_i dA_j \quad (1)$$

$V_{i,j}$ is the visibility function and the rest of terms are geometric magnitudes. Since it is not possible to find an analytical solution, the integral is solved by approximation. The importance of the form factor speed up calculation lies on the high cost of this process within the radiosity method (90%) and its apparent complexity $O(N^2)$.

In this work we compile, classify and evaluate different methods for the computation of the form factors, in particular, those based on the use of a single plane projection. It is also introduced a new algorithm, the polygonal projection onto the base of the hemisphere method (PPBH), which is compared with their precedents. After the previous work review, the basis of the method is presented in section 3. In section 4, we give a description of the experiments between the studied methods, showing the results in section 5. We finish with relevant conclusions and the new opened fields for the future.

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2. PREVIOUS WORK

One of the most studied methods has been the hemicube algorithm introduced by Cohen and Greenberg [Cohen85]. They proposed to locate a hemicube over the shooting patch so that the other patches can be projected on its five faces. Each one of the faces take advantage of the Z-buffer to solve the visibility problem. So, each face is a pixel buffer where each cell has its individual form factor value, called *delta* form factor, so that if a polygon is visible from a pixel it increases its form factor in the corresponding delta-value.

Nelson Max [Max95] made a deep study about this method. Following the same scheme of hemicube, the method of the cubical tetrahedron appeared afterwards [Jeffrey91]. This technique solely uses three half faces of hemicube to project the scene.

Following the tendency of the reduction of the number of projection faces, Sillion and Puech [Sillion89] presented the single plane method. In that proposal the scene only project on one plane whose extension varies according to the energy which we want to gather. The distribution of the sampling points must be chosen in such a way the delta form factors of the cells was approximately equal.

In a later work Recker et al. [Recker90] developed the method of the single plane derived directly from the hemicube. The plane where the scene projects corresponds with the top face of the hemicube and its extension follows the same rule that the method of Sillion above. In this approach each cell has its own delta form factor. In the same work, they introduced the method of the extended single plane, developing the idea that in the central part of the plane greater precision is required.

However, these methods share a common problem of rectangular partition of the plane. Recently, Vivó et al. [Vivo01] have introduced a new cell distribution for the form factors calculation using a polar plane. They take advantage of the geometry coherence of the problem. Their method improves the quality of the image and it has a similar time cost than the same family of methods.

Some new methods have been based in the use of the projection onto the hemisphere or unit circle proposed by Gatenby and Hewit [Gatenby91] with a hemisphere discretization method when it is split in triangular regions with nearly equal areas, or the ray tracing strategy utilized by Doi and Itoh [Doi98] when a surface element is subdivided into small triangular patches. A solid angle criterion is used to guarantee accuracy; when the angle is larger than the user tolerance, recursive triangulation is applied.

In this paper we present a method to compute form factors projecting onto the base of the hemisphere. It is based in the Nusselt analog. We will show how the computing time and the quality of the results can be better than other methods.

3. DESCRIPTION OF THE METHOD.

The Polygonal Projection onto the Base of the Hemisphere (PPBH) method computes the form factor integral measuring the area covered by each visible polygon projected onto the unit circle. This area is computed by a double projection: first onto the hemisphere and then orthogonally down onto its base. The relative area occupied accounts exactly for the form factor so we can expect an accuracy improvement instead of using rectangular techniques.

Next we present a geometrical description of the PPBH method. After that, we describe two different approaches to compute the projected area: a discrete and the continuous algorithm.

Base of the method

We use the central point of the shooting patch as origin of a local coordinate system and we transform each vertex to this reference system.

Now we have to project each edge onto de hemisphere and then onto its base. In order to do the double projection in one step, we only evaluate the X,Y coordinate of the intersection between the plane passing through the origin and the edge and the hemisphere (Figure 1). The intersection with the hemisphere will be a circle arc and its orthographic projection an ellipse arc.

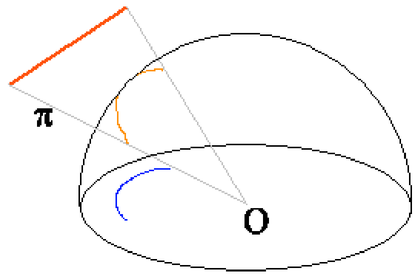

Figure 1. Edge projection onto the hemiesphere base

Being the imaginary plane that passes through the edge, $\pi : Ax+By+Cz=0$, and the formula of the hemisphere, $x^2+y^2+z^2=1$ $(z>=0)$, the final equation for Y coordinate of an edge projected point would be:

$$y = \frac{-ABx + C\sqrt{-x^2(A^2+B^2+C^2)+B^2+C^2}}{B^2+C^2} \quad (2)$$

Equation 2 is a curved line easy to calculate and it gives the exact value of the edge projection in the base. The projected vertex of the segment are the final values of the curve. Obviously, if we do the same for each polygon edge we will obtain a list of a linked curve segments. The region closed by the segment list is the polygon projection. According to the Nusselt analog the area of this projection relative to the hemisphere base area divided by π is exactly its form factor.

Discrete method

Given that equal areas on the hemisphere base correspond to equal form factors, we can superpose a square grid where each cell would account the same form factor. So, only knowing the length of the base cell, the delta form factor would also be known and no further storage. In fact, whatever decomposition is valid if all cells have the same area. We can control the cell size to adjust the precision of the computation. Each cell will be a sample point.

We have chosen a square grid covering the unit circle. Each element in the grid accounts the same form factor which could be easily calculated.

That square grid is used as a Z-Buffer where we can store the current depth and polygon id. In such a way the visibility problem is solved processing every polygon at a time. The ids of visible polygons are eventually available in the buffer when the whole scene had been processed. Thus, the form factor value of each polygon is approximated by the number of own cells. It must be notice that the irradiation is totally computed in one projection step because of all polygons are projected onto the hemisphere base. The visibility problem is solved by storing the distance between the nearest projected point and origin.

We can balance the error/speed ratio with the grid resolution. We follow the geometry of the problem so it is expected to improve accuracy on. However, the method shares with the hemicube the orientation and aliasing drawbacks.

Continuous method

When a polygon is projected onto the base of the hemisphere, the exact value of its form factor can easily computed. As it is known, we can calculate the area of a polygon in two dimensions with the sum of the area below each edge, and considering positive or negative sign according to the sense of the edge. We apply the same theory to compute de area of the projection, but now each edge is a curve segment. We compute the area below the curve edge by Eq. 2 integration. The limits of the integral are the projected vertex. The sign of an area depends on the sense the edge is traveled. Finally, the sum of those areas gives the area inside the projection.

The integration of Equation 2 corresponds with the following function:

$$Area = \left[\frac{1}{2\kappa}(\Phi + \Psi + \Omega) \right]_{x0}^{x1} \qquad (3)$$

where

$$\Phi = C \cdot arcsen(x\kappa / \eta)$$

$$\Psi = x\kappa\eta^2 C\sqrt{-x^2\kappa^2 + \eta^2}$$

$$\Omega = -x^2\eta^2\kappa AB$$

$$\eta = \sqrt{B^2 + C^2}$$

$$\kappa = \sqrt{A^2 + B^2 + C^2}$$

The integral must be calculated between the end points [$x0$,$x1$] of the edge projection if the curve doesn't change its direction. If there are two spans with different directions then the integral splits in two, one from $x0$ to xc and the other from xc to $x1$, being xc the minimum or maximum x of the curve.

Since we want to get the exact value of the form factor, it is no useful the Z-Buffer due to approximate sampling strategy. Therefore we need an exact algorithm to give solution to visibility problem. Traditionally, the problem has been solved in the object space by means of a kind of algorithms known as area-sorting algorithms. Those algorithms maintain a list of visible parts of polygons clipping new polygons against the visible list.

In addition, we have built a BSP tree structure to ensure next polygon resides behind the visible graph. In such a way the whole process speeds up.

At the end of the algorithm, we only have to traverse the graph projecting polygon pieces. The sum of the areas of the projected pieces is the visible polygon projected area and the form factor between these two polygons is directly computed.

4. IMPLEMENTATION AND RESULTS

Now we show a comparative study between our discrete PPBH and other projective methods. We have implemented the hemicube, the simple plane, the discrete and continuous polar plane as well as the discrete method presented above. In this way, it has been possible to compare computing time and image quality among these different methods changing the number of sampling points.

A series of test scenes has been chosen to experimentally observe the advantages and disadvantages of the method. The firts scene is a office room with one light and 4428 elements (plate 1) . And the most complex scene is a cloister with one big light and 80884 elements (plate 2).

It has been used the original implementation of Helios [Helios]. Any hardware acceleration has not been used so that the comparisons are more similar.

In order to obtain the error between the calculated scene and the real one, the method of the continuous projection onto the hemisphere base has been utilized as the base pattern. We have computed the average error for all the coefficients of the form factor matrix.

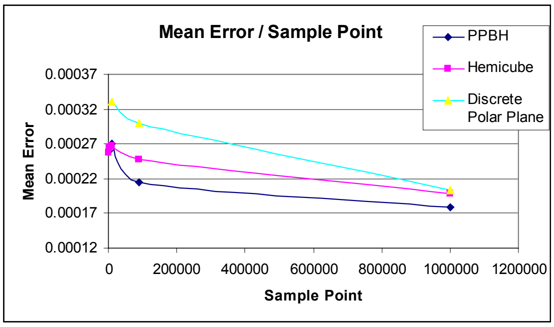

Figure 2. Mean error for the office scene (plate 1)

When the scene becomes more complex, our solution is clearly better than other discrete methods like the hemicube or the simple plane.

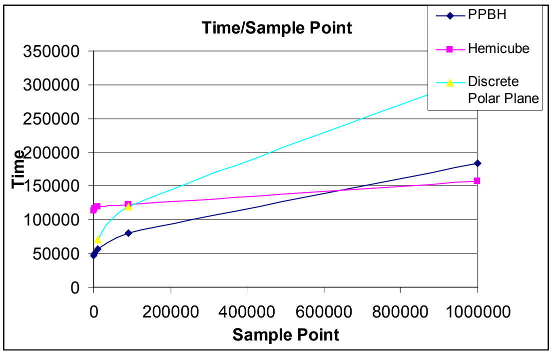

Figure 3. Computing time (ms) for the Cloister scene (plate 2)

In figure 3 we can observe how the new mothod computes form factors more quickly than hemicube with a number of sample points less than 650000 and it is always better than polar plane.

5. CONCLUSIONS AND FUTURE WORK

After making the different experiments and as a result we could summarize the advantages of the method of the discrete projection of polygons onto the hemisphere in the following ones:

- It does not need to store a delta form factors table, since it has a unique value for all cells.
- We can control the error changing the cell size.
- The projection is carried out in one step.
- The linear cost of the discrete PPBH is comparable to the other similar methods of projection but it has a better quality value.
- The PPBH takes in account the 100% of energy.

In summary, we have presented a new algorithm to compute form factor improving the accuracy of previous methods. One variant allows us to use it as exact solution and the other is compared with their precedents in speed and quality.

In order to improve the speed of the method, it is possible to take advantage of the hardware implementation of the Z-Buffer in the future.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

[Cohen85] Cohen, Greenberg. The Hemicube: A Radiosity Solution For Complex Environment. Computer Graphics, Vol 19(3), pp. 31-40. July 1985

[Doi98] Akio Doi and Takayuki Itoh. Acceleration Radiosity Solutions through the use of Hemisphere-base Formfactor Calculation. Jour. of Visualization & Computer Animation, vol.9(1), pp.3-15, 1998.

[Helios] HELIOS. www.helios32.com. Radiosity: A Programmer's Perspective, John Wiley & Sons, New York, NY, ISBN 0-471-30488-3

[Jeffrey91] Jeffrey C. Beran-Koehn and Mark J. Pavicic. A Cubic Tetrahedra Adaptation Of The Hemicube Algorithm. In James Arvo, editor, Graphics Gems II, pages 299-302. Academic Press, Boston, 1991.

[Gantenby91] gatenby N. Y hewit W.T. A proposed Alternative To The Hemicube Algorithm. Eurographics Workshop of Rendering. 1991.

[Max95] Nelson Max. Optimal Sampling Of Hemicubes. Procedings of the 4th Eurographics Workshop for Rendering. EG93RW, ISSN 1017-4656, pp. 185 - 200, and Addendum, pp. 348 - 351.

[Recker90] Rodney Recker, David George, Donald Greenberg. Acceleration Techniques For Progressive Refinement Radiosity. Computer Graphics. Vol 24 (2), pp 59-66. March 1990.

[Sillion89] François Sillion y Claude Puech. A General Two-Pass Method Integrating Specular And Diffuse Reflection. Computer Graphics, Vol 23(3). pp. 335-344. July 1989.

[Vivo01] R.Vivó, M.J.Vicent, J.Lluch, R.Mollá, P.Jorquera, "Study of the form factor calculation by single polar plane", IAESTED Visualization, Imaging, and Image Processing, Marbella, 2001.

8. STUDIED SCENES

Plate 1. Office Scene

Plate 2. Cloister scene