Freeform modelling by Curve Features manipulation

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ABSTRACT
Freeform features (FFF) manipulation is an emerging research area in geometric modelling field, which aims to develop innovative shape modification paradigms, more focused on global shape than on local curve modification. A major hindrance to FFF is given by locality properties of spline curves features representation, which forces designers to apply a direct control points manipulation, where a global modification tool or features based shaping might be more useful. Level-of-details (LOD) representation of curves, introduced by multiresolution wavelet analysis (MRA), allows to overcome spline locality weakness by describing features as detail coefficients, which can be reused over other different curves. In this paper we present a global modelling tool, based on MRA properties, useful to capture and extract curve features. Features, stored as a simple set of coefficients, may be easily applied over other curves, operating a smart modelling action and returning a good-looking merge between the new curve and the re-used form feature.

Keywords
Wavelets, curves, freeform features, global modelling, feature reusing.

1. INTRODUCTION
Freeform modelling is, at present day, one of the most active area in geometric modelling which aims to provide various design tools for shape modification, both in commercial and non-commercial CAD systems. NURBS representation of curves and surfaces lead to define direct control points manipulation tool [Far93a], which is the primary freeform modelling instrument, but reveals its weakness in global modelling shape area. In this context, freeform features (FFF) manipulation [van02a] [Ver01a] is a key element of shape modelling with respect the design, modification and reuse [Ver01b] of curves and surfaces features. A control points geometric description of parametric features is not suitable to formulate a reuse technique, thus our work is focused to elaborate a more efficient tool, capable to improve designing performance and overcoming control points dependency. This tool moves from the Multi-resolution Wavelets Analysis (MRA) theory, applied to curve representation [Fil94a] in order to extract and manage features in a global manner.

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Multi-resolution analysis based on wavelet [Mal89a] found wide application in computer graphics area [Ama01a] [Elb95a] [Uns97a] [Sto96a], both in curves and surface modelling. Filkenstein and Salesin also suggested an MRA B-spline wavelets curves representation enabling different kind of editing operations. In the meantime Lounsbery [Lou97a] fixes multiresolution theory with subdivision representation of curves and surfaces, showing up the connection between wavelets and geometric modelling.

In this paper we propose an innovative global modelling tool, which allows the designers to extract geometric features from a B-spline curve and permits their transfer onto a different target curve. A curve feature can be made up, for example, of fast gradient change, presence of impulses or other components which specifically characterize a curve. The aim of our modelling instrument is to strengthen designers actions to extract curves characterization by hierarchical decomposition steps, isolating features as *wavelets coefficients*, also known as *details*. Once details are completely captured, designers can exploit them to change shapes of different target curves, operating on the control points polygonal in a global manner. Designers are then enabled to create a *features*

library and reuse it to model a new curve profile reproducing a selected feature.
The remainder of this paper is organized as follows. In section 2 we briefly outline multiresolution curves definition, following Finkelstein [Fil94a] and Stollnitz notation [Sto96a]; in section 3 we formalize our global modelling tools. Results and examples are described in section 4 and conclusions in section 5.

2. MULTIRESOLUTION CURVES REPRESENTATION

End-point interpolating B-spline wavelets

Basic requirements for a multiresolution analysis is a set of nested approximation space V^j, such that:

$$V^n \subset V^{n-1} \subset \dots \subset V^0, n \in Z^+$$

spanned by a raffinable basis functions $\{\phi_i^j\}$ (*scaling functions*). As the scale *j* increases, the scaling functions can be expressed in a finer resolution:

$$\Phi^{j-1} = \Phi^j \mathbf{P}^j \tag{2.1}$$

where $\{\phi_i^j\}$ are end-point interpolating B-Spline scaling functions and Pj is the refinement matrix which refine the parametric domain in 2j equaly spaced intervals.
Similarly, the B-spline wavelet space W^j can be defined as orthogonal complement of V^j and spanned by a vector base $\{\psi_i^j\}$ such that:

$$V^j = V^{j-1} \oplus W^{j-1}$$

Since W^j is a subspace of V^j ($W^{j-1} \perp V^{j-1}$), a matrix $\mathbf{Q}^j$ can be found to satisfy $\langle \Phi^j, \Psi^j \rangle = \mathbf{0}$:

$$\Psi^{j-1} = \Phi^j \mathbf{Q}^j \tag{2.2}$$

Equations (2.1) and (2.2) are often reffered as two-scale relations for scaling and wavelets functions. Matrices P^j and Q^j are called *synthesis filters* and define the *recovery process*.
On the opposite, the decomposition process is realized by another couple of *analysis filters*, obtained resolving the following linear system:

$$\left[\frac{\mathbf{A}^j}{\mathbf{B}^j} \right] = \left[\mathbf{P}^j \mid \mathbf{Q}^j \right]^{-1} \tag{2.3}$$

The set of matrices $[\mathbf{A}^j, \mathbf{B}^j, \mathbf{P}^j, \mathbf{Q}^j]$ defines the *filter banks* and let to formulate the multiresolution wavelet transform for a b-spline curve.

Features Manipulation

A B-spline curve, $S^n(x) = \Phi^n \mathbf{C}^n$, $x \in [0,1]$, with $2^n + 3$ control points, $\mathbf{C}^j = \left[c_1^j, c_2^j, \dots, c_{2^j+3}^j \right]$, is considered as the finest approximation of $S^n(x)$ at level *n*, can be the represented in its smoother levels, *j*<*n*, as follow:

$$\begin{aligned} \mathbf{C}^{j-1} &= \mathbf{A}^j \mathbf{C}^j \\ \mathbf{D}^{j-1} &= \mathbf{B}^j \mathbf{C}^j \end{aligned} \tag{2.3}$$

which define the decompostion steps, while the recovery process is formulated as:

$$\mathbf{C}^{j+1} = \mathbf{P}^j \mathbf{C}^{j-1} + \mathbf{Q}^j \mathbf{D}^{j-1} \tag{2.4}$$

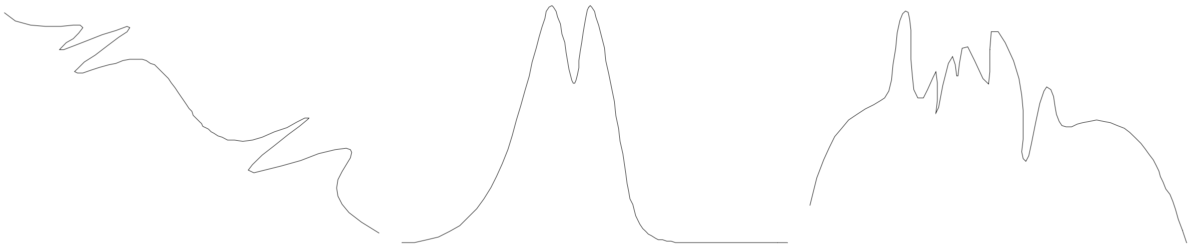
where $\mathbf{D}^{j-1}$ represents details coefficients also called wavelet coefficients.
In our work we are interested in details spaces which identify peculiarities owned by a source curve $S_s(x)$. Our attention will be pointed on the decomposition process of a curve with a particular feature that can be extracted simply isolating details $\mathbf{D}_s^j$ coefficients at each level of approximation. Details can be then exploited to store the shape as *geometric feature*.
Once a *source* curve is modelled, it is possibile to filter-out its macroscopic characteristic applying few decomposition steps, as defined in 2.3. The $\mathbf{D}_s^j$ coefficients can be then stored and adapted to a selected *target* curve $S_t^j(x)$ controlled by its $\mathbf{C}_t^j$, involving $\mathbf{D}_s^j$ in the refinement process defined as:

$$\mathbf{C}_t^{j+1} = \mathbf{P}^{j+1} \mathbf{C}_t^j + \mathbf{Q}^{j+1} (\mathbf{D}_s^j + \mathbf{D}_t^j) \tag{2.5}$$

The 2.5 defines the merging operation between details extracted from source feature and details which represent the original target shape onto which the new feature will be adapted.
Following this line of reasoning it will be possible to build a feature library, which enable designers to speed up and easily transfer a shape onto another significantly different, or simply use them as a global modelling action

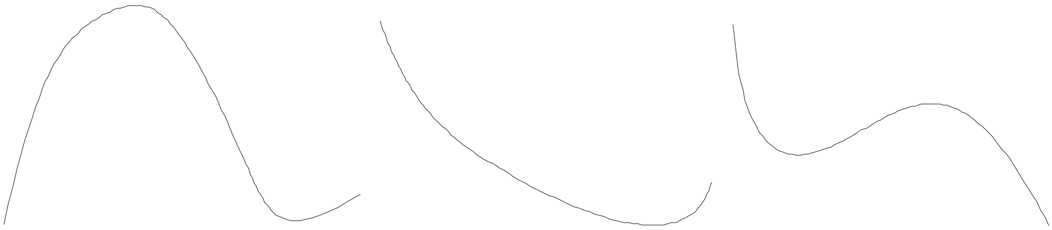
3. MODELLING EXAMPLES

In this section several examples of details extraction and their reuse and adaptation on different target curves will be illustrated. Source features (figure 1) are filtered-out from their most significant details parts which are then trasferred onto different kind target curves (figure 2). As defined by the (2.5), we obtained the extracted features adapted to new sweep (dashed curve) represented by target curves in figures 3, 4 and 5.



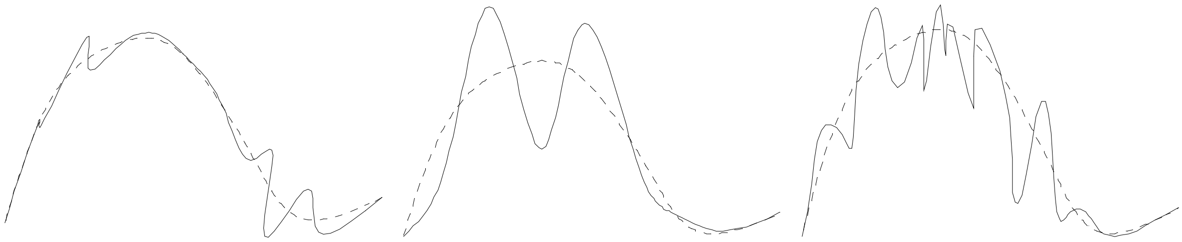
(a) (b) (c)

Figure 1: Source feature curves, (a) curve at level 4 with 19 control points, (b) curve at level 3 with 11 control points, (c) curve at level 5 with 35 control points.



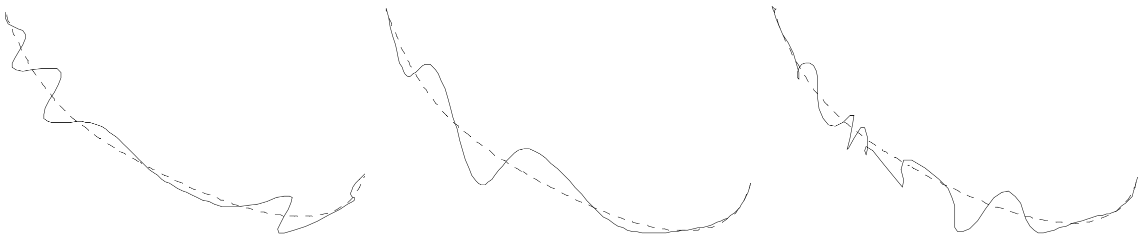
(a) (b) (c)

Figure 2: Different target curves, (a) curve at level 3 with 11 control points, (b-c) curve at level 1 with 5 control points.



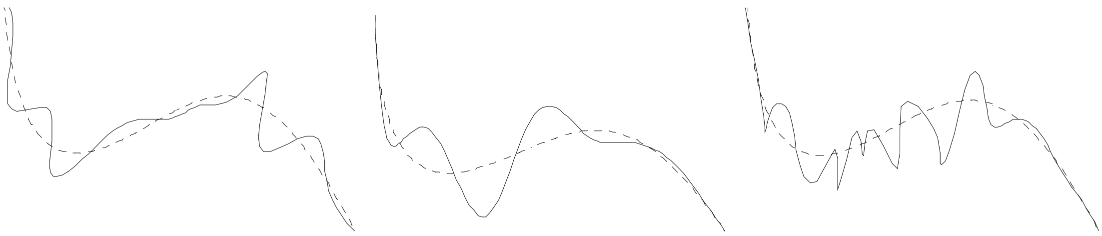
(a) (b) (c)

Figure 3: target curve in figure 2(a) with features extracted from figure 1.



(a) (b) (c)

Figure 4: : target curve in figure 2(b) with features extracted from figure 1.



(a) (b) (c)

Figure 5: target curve in figure 2(c) with features extracted from figure 1.

4. Conclusions

In this paper a multiresolution wavelet transform has been used as a geometric modelling tool for curves. We focus the attention on details manipulation; multiresolution wavelets transform reveals a desirable property which allows to extract details information from curves. Assuming details likewise a shape feature, multiresolution transformation has been used a modelling tool by exchanging details from different curves.

Truly, after implementation and several modelling tests, the extracted features from a source curve and the application of the same details to another curve, it has been proved that the described approach gives very interesting results. Also visual feedback has returned a good-looking final curve with clearly appreciable shape that remembers the old shape, but with applied new modifications evidently coming from source curve.

This property allow to formulate a special design tool for curves which can be easily integrated in a CAD modelling system, enabling designers to build their own *features library* and acting on a selected curve to change its shape in a global manner and opens to new ways of modelling instruments.

Final consideration have to be focused on computational costs regarding the entire multi-resolution process: recovery steps are governed by synthesis filters $\mathbf{P}^j$ and $\mathbf{Q}^j$ which typically have a banded structure allowing to design linear time algorithms.

A drawback, from the computational point of view, is represented by analysis filter $\mathbf{A}^j$ and $\mathbf{B}^j$ which rise the computational costs of decomposition to $O(m^2)$ -time, with $m = 2^j+3$, caused by their dense structure. Possible techniques to overcome this disadvantage are discussed in [Sto96a, War96a].

In conclusion, multiresolution wavelet analysis can be proposed as a global shape modelling tool thanks to its capability to extract features information from a source shape, enabling to start in designing new kind of global modelling instruments to integrate in modern CAD systems, increasing designers ability and designing quality.

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