

Skewed mirror symmetry in the 3D reconstruction of polyhedral models

A. Piquer Vicent
Universitat Jaume I
Dept. of Technology
E-12004, Castellon, Spain

pvicent@tec.uji.es

P. Company Calleja
Universitat Jaume I
Dept. of Technology
E-12004, Castellon, Spain

pcompany@tec.uji.es

R.R. Martin
Cardiff University
Dept. of Computer Science
PO Box 916, Cardiff CF24 3XF, UK

Ralph.Martin@cs.cf.ac.uk

ABSTRACT

We aim to reconstruct three-dimensional polyhedra from axonometric line drawings. Existence of mirror symmetry in polyhedra can assist the reconstruction process. We present a new approach for determining planes of mirror symmetry of such polyhedral objects based on prior detection of their planar faces and any axes of symmetry of these faces. The axes are obtained from skewed facial symmetries, for which we also give a new method of determination.

Keywords

Planes of Symmetry. 3D Reconstruction. Mirror Symmetry. Skewed facial-symmetry. Axis of Symmetry. Sketch Input. Sketch Understanding.

1. INTRODUCTION

Freehand sketches still constitute a fundamental tool for the engineer to express his or her creativity [Ull02], because CAD systems are still not entirely suitable for conceptual design. *Three-dimensional geometrical reconstruction* can help overcome this situation by recovering three-dimensional geometrical or perceptual information automatically from a sketch or two-dimensional line drawing.

Symmetry is a fundamental concept which human visual perception utilizes [Pal99]. Furthermore, many objects constructed by humans are symmetrical, because this makes them easier to interpret and manufacture, and because of functional requirements. Studies [Lan01] have shown that a large proportion of industrial components have some symmetry. As happens with other fundamental perceptual concepts, it is difficult to formulate visual symmetry in algorithmic terms, and it has to be broken down into simpler elements called perceptual cues or features.

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WSCG'2003, February 3-7, 2003, Plzen, Czech Republic.
Journal of WSCG, Vol.11, No.1, ISSN 1213-6972
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In the mirror symmetry of polyhedral objects, the most important perceptual cue is *skewed facial symmetry* [Kan81], which arises from the affine transformation that symmetric faces of symmetric polyhedra undergo under parallel projection. Only mirror symmetry is considered in this work, since other symmetries (like rotational) require an entirely different approach.

In this paper we present a novel method for determining the skewed planes of symmetry of polyhedral objects, starting with a two dimensional projected axonometric line view. The method is based on finding axes of facial symmetry and other perceptual cues for the candidate planes of symmetry. To restrict the size of the search space, the method discards candidate planes which present contradictory cues or which have a low probability.

Our method is more efficient than prior approaches such as Friedberg's [Fri86] and Grimstead's [Gri95] for finding the axes and for removing the associated skew. Our method is implemented in a system that automatically reconstructs a large variety of polyhedral, origami and wire-frame objects [Com99]. The problems of going from free-hand drawings to line drawings have been solved elsewhere [Varl00c]. Hence, it works for freehand drawings, as well as accurate projections, since it is tolerant to imperfections in the original drawing.

2. PREVIOUS WORK

Skewed symmetry of faces is the main cue for mirror symmetry in polyhedral models. All symmetric faces of the polyhedron (such as the face on the left of Figure 1) appear as oblique or skewed symmetric faces after projection (Figure 1, right).

Algorithms exist that detect symmetry or *approximate* symmetry, either by starting out with a finite set of points like Alt [Alt88] and Mills [Mill01], or by using approaches like Highnam [Hig86], who represents the points by a string of characters, and tests whether it is palindromic. Algorithms have also been devised that stem from a plane figure made up of a set of elements, all of which are included within a list of geometrical patterns [Ata85]. Marola [Mar89] has focused on detecting axes of symmetry in plane figures obtained from digitized greyscale images, where noise arising during contour detection was important. Other studies such as [She99] analyze how to compute symmetry and measures of the degree of symmetry in polygons.

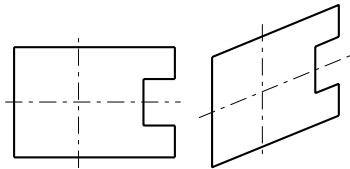


Figure 1. A symmetrical figure and its skewed symmetry after projection

We assume that each input drawing is (approximately) the projection of a polyhedral model onto a plane. After projection, the polygonal faces bounding the polyhedron are represented in 2D by contours which are deformed with respect to the shape they had in 3D (again, see Figure 1). A face that is symmetrical in 3D has that symmetry deformed during 2D projection. Thus, if we wish to find original symmetries by considering projected polygons, we must study what happens to symmetry during such transformations. The result is *skewed symmetry*.

Kanade [Kan81] classified skewed symmetry as a regularity and made the following statement about it: “A skewed symmetry depicts a real symmetry viewed from some (unknown) view direction.”

In his study, because of his particular 3D reconstruction method, Kanade approached skewed symmetry as producing a constraint that must be applied within the gradient space, the space of possible normals for the face. He showed that the skewed symmetry represents a real symmetry if the face gradient lies on a hyperbola and, in addition, it minimizes the slant of the face. (His input drawings did not have to be closed and did not show the hidden

parts of the objects—parts of the object were not visible.)

However, it was Friedberg [Fri86] who tackled the issue in a comprehensive manner. He gave a clear formulation for searching for potential axes of skewed symmetry, i.e. axes of symmetry after projection, and for checking whether they correspond to real axes of symmetry before projection or not. In order to do so, he formulated a *deskewing function* T which mapped the projected axes back into a hypothetical original orientation; depending on the figure obtained he decided whether to accept the potential skewed symmetry axis as an axis of symmetry of the original surface. He gave a necessary, but not sufficient, condition for an axis to be a skewed symmetry axis.

Others, such as Grimstead [Gri95], have also followed the steps of Kanade and Friedberg in the search for skewed facial symmetry using gradient space in an attempt to reconstruct three-dimensional drawings.

Oh [Oh88] gave a method for finding the skewed symmetry axes of polygons in order to determine surface orientation. Oh’s method is simple but has the disadvantage of being extremely time-consuming.

In [Gro91], Gross classified skewed symmetry methods into two types: global and partial. Local methods compare pairs of points on the contour. Such local methods have the advantage over global methods in that they do not require consideration of the whole contour, which is important when processing drawings with hidden lines removed, for example. However, the skewed symmetry processing method developed by Gross [Gro94] is based on prior knowledge of how many axes of skewed symmetry the contour has, and this number cannot be greater than five.

Yip [Yip94] gave a method for finding the axes of skewed symmetry using an elliptic Fourier descriptor and arc-length parameterization, enabling reconstruction of the symmetry from a parallel projection of the faces of a polyhedron.

Some authors have studied skewed symmetry of shapes with curved contours [Pos92]. In most cases the input comes from real images in which hidden parts cannot be seen, and for which contour recognition was needed. In such cases there can be a lot of noise in the input data [Gro91], [Sug94], [Cha95], [She01], [Sau01]. Others take measurements from real objects [Zab94], for which certain characteristics are already known.

Wen [Wen95] followed largely in Friedberg’s footsteps and gave a simple method for finding which are the critical points on the contour where symmetry

should be checked, and which could be points the axes might pass through.

Uluvar and Nevatia [Ulu93], [Ulu95], used symmetry as an important property in the reconstruction of 3D models; their study in [Ulu95], however, focuses on surfaces of revolution.

In the paper by Lipson and Shpitalni [Lip96], a very reduced formulation was presented, which did not attempt to find out the exact number of axes of skewed symmetry or their exact positions. Their work was aimed mainly at reconstruction, and on how to calculate the probability of a given planar face having a possible axis of symmetry.

3. DETECTION OF FACES

Detection of skewed facial symmetry requires the prior determination of the faces of the two-dimensional image. This is a potentially complex problem because the faces of a polyhedron are projected as polygons, but these polygons intersect each other where the projections of the visible and hidden faces overlap (Figure 2). Methods exist to resolve this problem [Cou86] and [Shp96], at least in simple cases. After detecting the faces, we can find the possible axes of symmetry for each face.

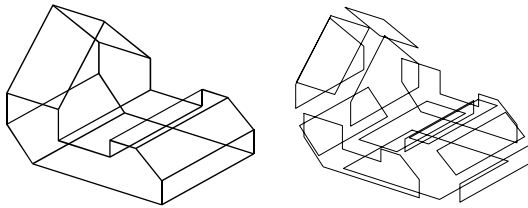


Figure 2. Projection of a polyhedron with transparent faces and their detection.

Line labelling methods for line drawings can also be used as a precedent to face detection (from Huffman [Huf71] and Clowes [Clo71] to Varley [Var00a], [Var00b]). Relatively little work has been concerned with the detection of faces in line drawings containing *hidden* edges. Two methods developed by Leclerc [Lec92] and Shpitalni [Shp96] are noteworthy. In both methods, face detection is divided into two stages: a) detection of cycles of edges that are candidates to be faces of the object, and b) selection of true model faces from the cycles.

We are also currently working on a method for automatic determination in two dimensions of faces of polyhedral objects represented by an axonometric projection or conic perspective. The method is based on graph theory and perceptual cues. Initial results are promising, and show low computational cost, but more work is needed before our algorithm is fully developed.

4. FACIAL SYMMETRY

As human observers, we are sensitive to symmetry. Thus, we try to interpret images as being derived from symmetrical models. The *Law of Symmetry* [Pal99] states that we interpret an image in such a way as to produce a model that is as symmetrical as possible. Geometrically speaking, symmetry is a geometric transformation of space that leaves shapes unchanged. For the purposes of this paper, we only consider *mirror* or bilateral symmetry, where *reflection* in a mirror or symmetry plane leaves the object unchanged.

In order to try to determine three-dimensional symmetry, we start by looking for symmetries of the 2D faces of the polyhedron. This section considers how to obtain all axes of facial symmetry depicted in the 2D image. The idea of skewed facial symmetry is outlined in Subsection 4.1, following [Fri86], and in Subsection 4.2 our own approach to detecting it is explained.

4.1. Skewed Facial Symmetry

When the 2D points of a projected symmetric figure are represented in 2D skew coordinates, the coordinates have a skewed symmetry. In a sense, skewed symmetry is a “generalized” symmetry, and we can consider orthogonal symmetry as a special case where the axes make an angle of 90° .

Mirror symmetry in 2D is determined by an axis lying in the plane of the figure that divides it in two parts, such that for each point on one of the parts, there is a corresponding point on the other [Wey51]. Both points must also be equidistant from the axis and determine a straight line that is perpendicular to it (see Fig. 3- left). Thus, a set of points in the Euclidean plane has bilateral symmetry if there is a line, the axis of symmetry, such that the set is invariant under reflection in this line.

Skewed facial symmetry occurs when, after carrying out a parallel projection (which can be either oblique or orthogonal to the plane of projection) of a symmetric polyhedral object, certain faces have one or more axes that divide them into two parts such that for each point on one of the two parts there is a corresponding point on the other. Both points must be equidistant from the axis and determine a straight line making a conjugate direction to the axis. This conjugate direction is the same for all points on a given face. The conjugate direction is the projection of the direction orthogonal to the un-projected symmetry axis. In Figure 3-right, U_I is the axis of skewed facial symmetry and V_I is its conjugate direction.

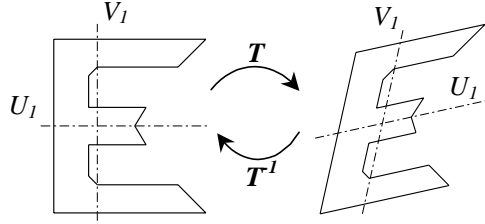


Figure 3. Simple face with mirror symmetry

Thus, a set of points in the Euclidean plane has skewed symmetry if there is a line, called the axis of skewed symmetry, and a fixed angle, such that the object is invariant under the transformation which replaces each point of the object by a corresponding point on the opposite side of the axis, an equal distance from the axis, lying along the line which makes the given angle with the axis.

We define an orthogonal X - Y coordinate system for each face of the polyhedron (see Figure 4), such that (x_p, y_p) are the coordinates of each point P which is a vertex of the polygon after projection of the face. We also define a system of U - V axes of skewed symmetry where the u coordinate is measured along the axis of skewed symmetry, and v is measured along the transverse axis, in the conjugate direction. If we call the angle of rotation between the X axis and the U axis α , and the angle between the transverse axis and the X axis γ , the skew is given by $\beta = \gamma - \alpha$, where α and γ lie in the interval $[0, \pi]$. It has been shown [Fri86] that these angles can be calculated from the orthogonal coordinates of the N vertices of polygon, using what is called the *Fundamental Condition of Symmetry*, as follows:

$$\alpha = \arctan \left[\frac{\sum_{i=1}^N x_i y_i \tan \gamma - \sum_{i=1}^N y_i^2}{\sum_{i=1}^N x_i^2 \cdot \tan \gamma - \sum_{i=1}^N x_i y_i} \right] \quad (1)$$

$$\gamma = \arctan \left[\frac{\sum_{i=1}^N \frac{y_i^2}{\tan \alpha} - \sum_{i=1}^N x_i y_i}{\sum_{i=1}^N \frac{x_i y_i}{\tan \alpha} - \sum_{i=1}^N x_i^2} \right] \quad (2)$$

Once the condition of symmetry has been used, we can deduce the skewing function T , which is a linear geometric transformation mapping deskewed polygonal contours into their skewed versions after projection:

$$P \cdot T = Q$$

wherein,

$$P = [x, y], \quad Q = [u, v].$$

$$T = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \cos \alpha \cot \beta - \sin \alpha & \sin \alpha \cot \beta + \cos \alpha \end{bmatrix} \quad (3)$$

Its inverse provides us with the function we really want, the deskewing function which maps the skewed polygon back into its deskewed original appearance (see Figure 3):

$$Q = T^{-1} P$$

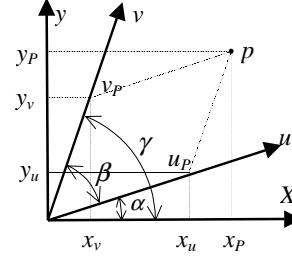


Figure 4. Cartesian and skewed coordinates

where

$$T^{-1} = \begin{bmatrix} \sin \alpha \cdot \cot \beta + \cos \alpha & -\sin \alpha \\ \sin \alpha - \cos \alpha \cdot \cot \beta & \cos \alpha \end{bmatrix}. \quad (4)$$

4.2. Detecting Skewed Facial Symmetry for Simple Polyhedra

The centroid of any closed contour does not change its position under operation of the skewing function or the deskewing function; both the skewed symmetry axes and symmetry axes pass through the centroid. Thus we use this point as the origin of the coordinates.

We can find the axes of skewed symmetry by taking advantage of the fact that, since we are dealing with polygonal faces, the only possible axes (candidate axes, such as U_1 and U_2 in Figure 5) must pass through one of the vertices of the polygon (v_i) or through one of the mid-points of its edges (m_i), as well as through the centroid (c). Furthermore, if the polygon is symmetrical, the number of axes of symmetry can be at the most the same as the number of vertices.

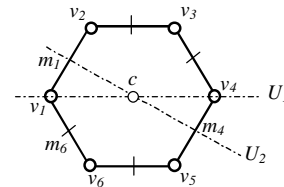


Figure 5. Tentative face symmetry axes

The pairs of points formed by the centroid and the successive vertices or mid-points of half the contour determine all the candidate axes (for example v_1 and c for U_1). From each candidate axis of symmetry, we

know α and by applying Equation (2) we obtain the value of γ and hence the direction of the candidate conjugate axis. Thus, the process for determining skewed facial symmetry for a given face consists of repeating the following over all candidate skewed symmetry axes:

1. A face is chosen and its centroid is calculated.
2. Some vertex (or edge mid-point) is chosen, which determines the candidate axis.
3. Given v_i , we know the value of α , and γ is calculated from the *Condition of Symmetry*. Thus, we now have a possible axis of symmetry and its conjugate direction.
4. Using the values of α and γ we now calculate the matrix for the deskewing function T^1 using Equation (4), and apply it to all the vertices of the contour. This will provide us with a supposedly symmetric face without skew.
5. We check whether the resulting face has mirror symmetry.

To check whether the face we obtain is symmetric, we might try to apply the condition $\sum_{i=0}^{N-1} x_i y_i = 0$.

However, this condition is necessary but not sufficient in itself for a polygon to be symmetric.

However, we have formulated the deskewing function T^1 in such a way that, if the figure is symmetric after the transformation, it will also be symmetric with respect to the X axis. Therefore, the following *Condition of Total Symmetry* should also be satisfied:

$$\begin{aligned} x_{k-i} &= x_{k+i} & (5) \\ y_{k-i} + y_{k+i} &= 0 \end{aligned}$$

where k is the (cyclically numbered) vertex or edge mid-point the axis of symmetry passes through, and $i = [1, 2, \dots, n/2]$.

For each closed polygonal contour, this process is applied n times (to $n/2$ vertices and $n/2$ mid-points of edges): half the contour gives all the possible skewed symmetry axes the contour can have.

Note that in Equation (5) a tolerance must be applied because the skewed figure may not be precise, due to the origin of the drawings. For instance, if the drawing is a sketch it is not necessarily very accurate: while we assume it contains all the elements of the solid to be reconstructed, it does not have to be a perfect projection of the object it represents. Thus we must decide upon standards of acceptability when deciding if a polygon represents a skewed symmetry. In our *REFER* [Com99] application we allow a 10%

error in the positions of coordinates determining the contour as our tolerance criterion:

$$|x_{k-i} - x_{k+i}| < 0.1 * |x_{max} - x_{min}| \quad (6)$$

$$|y_{k-i} + y_{k+i}| < 0.1 * |y_{k-i} - y_{k+i}| \quad (7)$$

With this last check, the process of detection of skewed axes of symmetry for each polygon face is finished. All possible axes of every face of the polyhedron are detected as shown in Figures 6, 7, 8 and 9.

5. PLANES OF SYMMETRY IN POLYHEDRA

Given the faces detected in the line drawing, and the axes of symmetry detected for each face, it is now possible to search for a closed sequence of lines (polygon edges plus symmetry axes) lying in a single three-dimensional plane of symmetry. Each such line ends either at a vertex, or a mid-point of an edge.

We perform our computation in two-dimensional space, so that the resulting output comprises *skewed planes of symmetry*: real planes of symmetry viewed from some (unknown) viewing direction.

As noted, each skewed plane of symmetry is made up of skewed axes of symmetry and edges of various polygons. Consecutive lines (axes or edges) must belong to different faces of the graph representing the polyhedron after projection. This is because if two consecutive lines belonged to the same face, this face itself would have to lie in the plane of symmetry.

If a closed sequence of axes and edges that meet each other can be found, thus giving rise to a planar polygon, then the plane that contains that polygon is considered to be a plane of symmetry.

However, the number of candidate lines belonging to any single plane of symmetry is large. For instance, Figure 6, a line drawing with 34 edges of polygons plus 35 axes of facial symmetry found with our method, has 69 candidate lines. Searching from such a high number of lines for a closed circuit lying in a plane of symmetry is very time consuming. Consequently we use various rules to reduce the number of candidate lines.

The three main rules are:

- When a symmetry axis across a face ends in the mid-point of an edge, the opposing face on the other side of the edge must also have an axis ending in the mid-point of this edge.
- Axes of symmetry of a face going through a vertex can continue along an edge or another axis of symmetry. However, two consecutive lines must not belong to the same face. Consequently, if one axis ends in a vertex there are two acceptable possibilities:

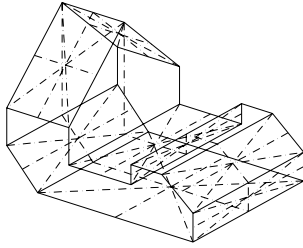


Figure 6

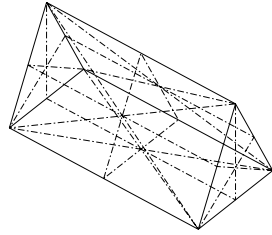


Figure 7

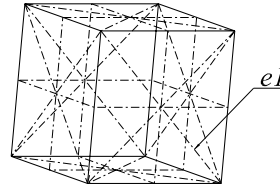


Figure 8

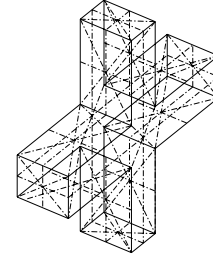


Figure 9

Figures where each face has all its axes of skewed symmetry detected.

- some other face adjacent to this vertex has an axis ending in the same vertex,
 - the chain of lines forming the plane of symmetry can continue along any edge (e_1) that ends in the vertex but does not belong to the face having the first axis of symmetry. At the other end of the edge another axis (a_2) of symmetry must meet it. This must belong to a face different to any other face sharing the edge, and obviously also the first face (See Fig.12).
- Suppose two consecutive candidate lines l_1 and l_2 have vertices v_1 and v , and v and v_2 . Suppose furthermore that v_1 and v_2 belong to the same face. Because the skewed plane of symmetry must be represented by a closed chain of lines (axes or edges), a line connecting both vertices (v_1 and v_2) must be present for that face if l_1 and l_2 do belong to a symmetry plane. For instance, in Figure 10, axes l_1 and l_2 can not belong to a single symmetry plane because there is no line between v_1 and v_2 on the face to which they both belong. This observation can also be extended to larger circuits of lines.

Any line or axis not meeting all of the above requirements is discarded from the candidate lines which can lie in a plane of symmetry. In many cases, discarding one axis results in being able to discard a chain of axes for a plane of symmetry. However, even with these reductions, we will often still have too many candidate lines. For this reason we use another extra heuristic condition, chosen after processing a large number of sketches and finding that it gives good results in general.

- We eliminate candidate axes of symmetry connecting two vertices across a single face.

(Note that this rule is *not* applied to *edges* of faces, only axes *crossing* faces.) With this extra rule, we usually obtain satisfactory results in our search. In only a few cases (e.g. axis e_1 in Fig.8) does applying this rule cause some real planes of symmetry to be

missed. This issue will be discussed in the next Section.

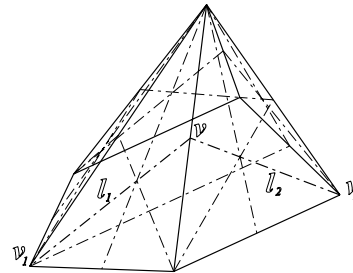


Figure 10. Graph with skewed facial axis of symmetry detected

6. LIMITS IN THE SEARCH FOR PLANES OF SYMMETRY

The method explained in Section 5 gives very good results with a low computational cost for more than 150 sketches evaluated. However, it has some limitations.

Firstly, we note that our method for searching for planes of symmetry in two-dimensional graphs depends directly on the correct detection of faces in a graph, and correct detection of axes of facial symmetry on them. Although these are generally well-understood problems, as discussed in our earlier references, any incorrect behavior of these methods will affect our method.

Secondly, the most important limitation is caused by our last rule to eliminate further candidate axes for planes of symmetry. This rule is very useful in the majority of examples, but it can eliminate some real planes of symmetry. For instance, a cube has 9 planes of symmetry, but applying the last rule causes our method not to find 6 of them. However the program still finds the three most obvious planes of symmetry. It is reasonable to say that these are the three main planes of symmetry for a cube. Therefore, the method detects all relevant and most important planes of symmetry (see figures 11 to 14).

The same problem occurs in Figure 9, where applying this rule, two planes of symmetry are missed ($s-q-u-v$ and $p-r-w-t$). But we find three planes of symmetry

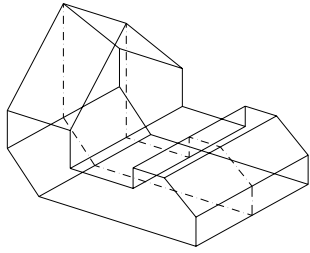


Figure 11

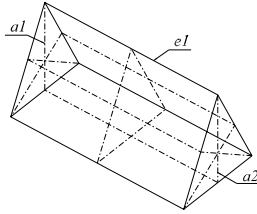


Figure 12

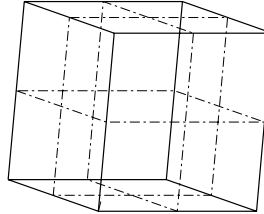


Figure 13

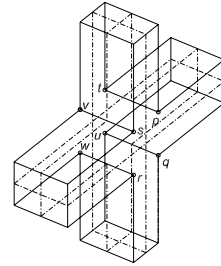


Figure 14

Figures with detected skewed planes of symmetry.

(see Figure 14), and psychologically could be considered the fundamental planes of symmetry of the model.

A third limitation is that the method does not find planes of symmetry that consist exclusively of edges of the polyhedron. Two pyramids joined by their bases form an example. Again, in such cases, we expect to find other planes of symmetry more significant than this one.

We thus feel that the limitations are not as important as they might seem. In the majority of cases where our last rule causes us not to find some plane of symmetry, there are other planes of symmetry that seem to be perceptually more significant which are detected with our method. Indeed, it is not easy in general to find examples with planes of symmetry which cannot be detected.

7. CONCLUSIONS

A new approach for detection of planes of skewed symmetry of polyhedra depicted by a two-dimensional graph projection has been presented. A separate contribution of this paper is an improvement to previous approaches for detection of axes of skewed facial symmetry of closed contours.

The ideas have been implemented in our REFER [Com99] software for the three-dimensional reconstruction of geometric models from sketches. We have obtained low processing times and a small failure rate (in terms of not finding symmetry planes which are present), thus representing a step forward in the reconstruction of 3D models.

On the other hand, a general approach to applying model symmetry has not yet been developed for reconstruction purposes. Our new method for detection of planes of symmetry has the potential to be applied to other steps of the reconstruction process. Furthermore, other downstream CAD/CAM processes may also be able to use any identified planes of symmetry—for example, finite element analysis need only consider part of the model if it is known to be symmetric.

We finally note that further work is needed to analyse rotational symmetries.

8. ACKNOWLEDGMENTS

This work was partially supported by *Fundació Caixa Castelló- Bancaixa* under the *Universitat Jaume I* program for Research Promotion (Project P1-1B2002-08, titled “From sketch to model: new user interfaces for CAD systems”), and by *Cardiff University*, especially to the *Computer Science Department*.

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