NONCONFORMITY PROBLEM IN 3D GRID DECOMPOSITIONS

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ABSTRACT

3D modelling and visualization based on general meshes needs a great amount of memory demands. Regular grids can substantially decrease them. Moreover, using the regular data structures results in efficient numerical procedures and simplicity of algorithms. On the other hand, disadvantage of regular grids is in lower flexibility of shape expression. One of the possibilities how to improve the shape expression of a regular grid is its decomposition to tetrahedra. The paper concerns the problem which arises here – *nonconformity of decomposition.*

Keywords: Structured mesh, Decomposition, Nonconformity.

1 INTRODUCTION

The decomposition of the space to polygons in 2D, resp. to polyhedra in 3D is very important step within the numerical modelling of any physical phenomena. Physically based modelling of complex phenomena often leads to *high performance computing problems.* So the reduction of memory demands and the proposal of efficient numerical procedures used in solver are very important. To achieve this, meshes with regular structure are suitable [Blaheta99]. We shall consider *structured mesh* which can be obtained as a result of

1. `the deformation of regular rectangular grid (i.e. we can shift the nodes of the grid but we cannot change coincidency of them),`
2. `decomposition of the cells to triangles in 2D, resp. to tetrahedra in 3D; here no new nodes are added to the resulting mesh.`

In the paper we want to describe the problem of *nonconformity of decomposition* which arise when we go from 2D to 3D mesh generation: *two elements of decomposition are conform if their intersection is either empty, or is a common face, or is a common edge or is a common corner. In the opposite case the pair of elements is nonconform. Decomposition is conform if each pair of its elements is conform.*

The paper is organized as follows. The 2nd section describes the 2D case of mesh generation. Here the most important properties of 3D decompositions are mentioned too. Next section describes the arising of nonconformities and in the 4th section it is shown how the nonconformities can be removed.

2 MESH GENERATION

In [Kolcun96] the following sequence for the structured mesh generation is proposed:

```
A. generate regular grid,
B. shift nodes due to geometry
   of domain,
C. indicate the diagonals due
   to geometry of domain,
D. add the rest of diagonals,
E. identify the decompositions
   of cells.
```

2D case of the sequence is illustrated in Fig.1.

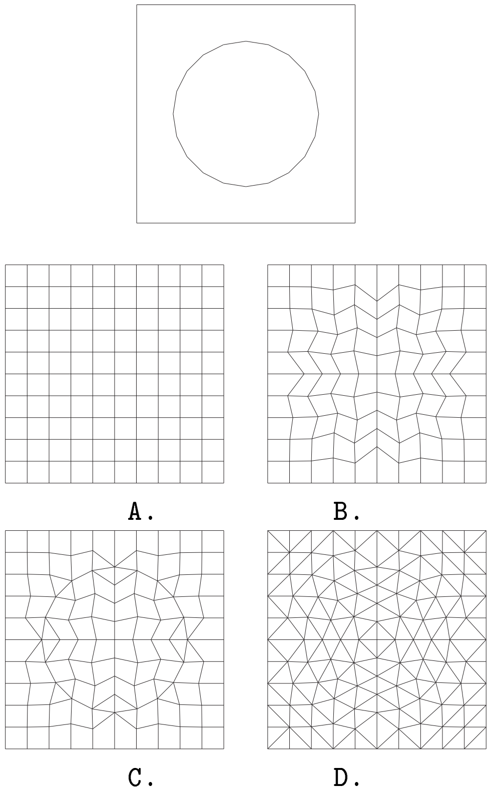


Figure 1: Required geometry of domain and steps `A.`–`D.` of the generation of structured triangulation in 2D.

Various approaches of grid deformation – step `B.` can be used, e.g. methods based on the solving the P.D.E., transport mapping (algebraic) methods, methods used general parametrisation, etc., [Grids91], [George91], [Thompson98]. In our case the strategy of grid deformation is based on averaging of the neighbour nodes of the grid [Kolcun94a], [Kolcun99b].

The main emphasis of the whole process of the mesh generation in our approach is in the steps `C.-E.` It is important that in 2D case **1.** there is only a restriction for diagonals in case of nonconvex tetragon – steps `C, D,` **2.** in any case of choice of diagonals we obtain the conform decomposition, **3.** decomposition of tetragon doesn't depend on decompositions of neighbour tetragons – steps `C, D,` **4.** decomposition to triangles with determined diagonal is unambiguous – step `E.`

As it is proved in [Kolcun94b],[Kolcun99b], the situation in 3D is much more difficult: **1.** there exist configurations of diagonals in the grid cell for which there are only nonconform decompositions – Fig.2a) **2.** there exist configurations of diagonals for which there are no decompositions to tetrahedra – Fig.2b), **3.** there are more different decompositions with the same configuration of the face diagonals of the cell, **4.** there exist 72 conform decompositions of cell to six, and two decompositions to five tetrahedra (decompositions to six tetrahedra are prefered due to preserve the regularity of data structures).

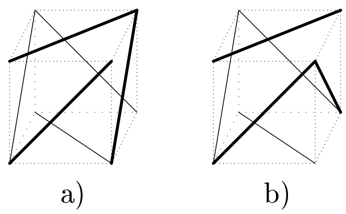


Figure 2: "Forbidden" configurations of diagonals: none node is incident to three diagonals and a) exactly one pair of diagonals in the pair of opposite faces is parallel, b) no pair of diagonals in the opposite faces is parallel.

In [Kolcun96] it is shown that a few of nonconformities don't spoil the iterative FE-solution when nonconform mesh is used. However, the ratio of the number of the configurations of type Fig. 2 a), b) is relatively high – it achieves 37.5% of all configurations of diagonals. So it is important to understand the mechanism of the nonconformity arising in the cell decomposition.

3 NONCONFORM DECOMPOSITIONS

The nonconformity between elements from different cells is denoted as a 'face' nonconformity. Nonconformity within one cell is 'inner' nonconformity.

There is proved in [Kolcun01] that all inner nonconform decompositions we can obtain in following way.

1. `We divide the cell to two 3-sided prisms.`
2. `The space diagonals from the cutting plane are joint to prisms - Fig.3.`
3. `Each prism with the space diagonal is decomposed to three tetrahedra - Fig.4.`

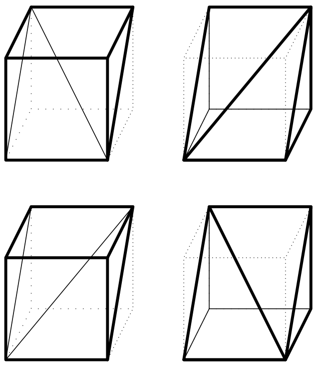


Figure 3: Two possibilities how the space diagonals can be joint to prisms.

The number of possibilities of these steps are

$$\mathcal{N}_1 = 6, \quad \mathcal{N}_2 = 2, \quad \mathcal{N}_3 = 3.$$

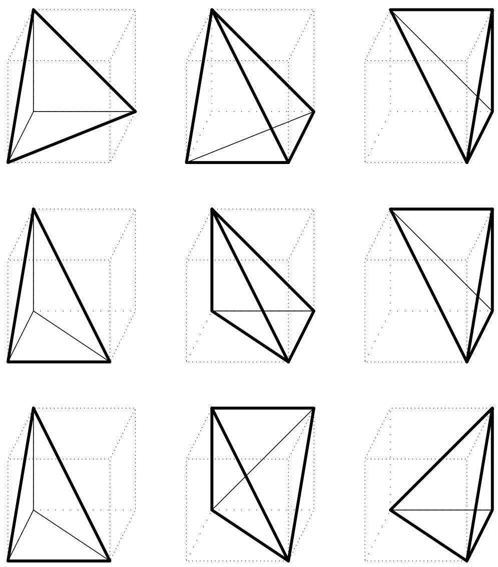


Figure 4: Three different decompositions of prism with the same space diagonal to three tetrahedra.

As the steps `1.-3.` are mutually independent, the number of all nonconfom decompositions is obviously

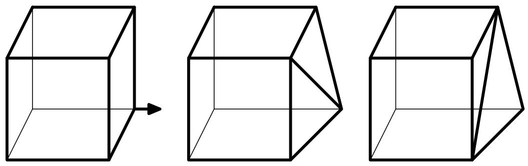
$$\mathcal{N} = \mathcal{N}_1 \times \mathcal{N}_2 \times {\mathcal{N}_3}^2 = 108.$$

4 DECOMPOSITIONS IN GENERALIZED CELL

Let us consider a little bit more generalized cell of mesh, which is obtained by deformation of orthogonal cell so, that none four vertices are coplanar. We replace each nonplanar face with the pair of triangles using the face diagonal from the original cell decomposition, Fig.5.

We can see obviously, that the face nonconformity is now replaced with new tetrahedron - Fig.6

The same mechanism can be used to remove inner nonconformity. So the number of tetrahedra in the decomposition of generalized cell vary from five to thirteen. (The last case we obtain from decomposition to six tetrahedra with nonconformity in each face and with inner nonconformity too.)



(a) (b) (c)

Figure 5: Generalization of the orthogonal cell – example of deformation of one vertex: (a) – orthogonal cell, (b) – convex face, (c) – nonconvex face.

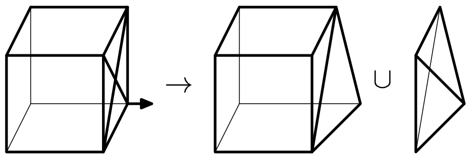


Figure 6: Arising of new tetrahedron when the cell with face nonconformity is generalized.

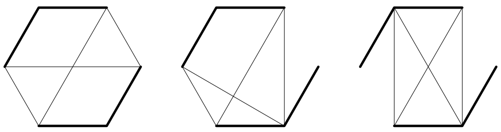
For better illustrativity the dual representation of decompositions is useful: graph $G(V, H)$, represents the decomposition

$$\mathcal{E} = \{e_i : i = 1, \ldots, n\},$$

iff

$$\begin{aligned} V &= \mathcal{E}, \\ H &= \{(e_i, e_j) \in V^2 : dim(e_i \cap e_j) = 2\}. \end{aligned}$$

Graph represents 'strong' neighbourhood of elements (i.e. the neighbourhood at least with common part of face).

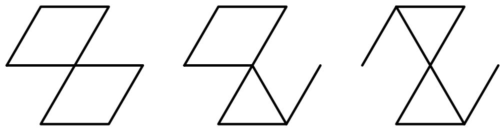


1. 2. 3.

Figure 7: Dual representation of cell decompositions with inner nonconformity. Pairs of nonconform elements are connected with thin edge.

For nonconform decompositions of orthogonal cell we obtain the representations from Fig.7.

When we use generalized cell, new tetrahedron appears and the decompositions which are obtained from Fig.7 have the representations from Fig.8.



1. 2. 3.

Figure 8: Dual representation of nonconform decompositions in the generalized cell without 'face' tetrahedra.

'Face' nonconformities add next tetrahedra into decompositions – Fig.9.

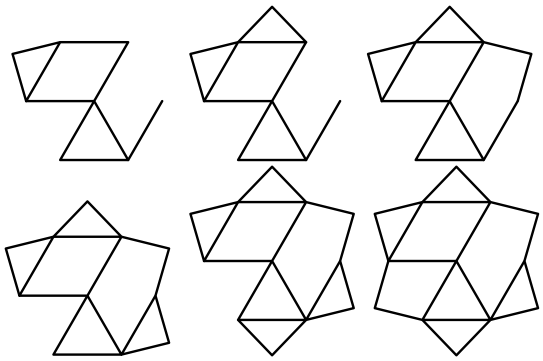


Figure 9: Dual representation of cell decompositions which are obtained from decomposition 2. from Fig.8 when 'face' nonconformities are added.

5 CONCLUSION

Generalization of decomposition of rectangular grid to tetrahedral elements enables to increase the flexibility of geometry expression. But the problem of nonconformity in 3D arises. It is possible to remove the nonconformity problem using the cell deformation approach but number of elements of decomposition increases. Hence, regularity of data structures is spoilt.

Future work will be focused on analysis of metric properties of structured decompositions [Korotov99].

Presented structured meshes are based on 'brick' grids. Effort will be done to analysis of the structured decompositions with different basis [Goldberg74], [Jucovič81] too.

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