IMAGE-BASED RENDERING AND GENERAL RELATIVITY

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ABSTRACT

Imaged-based rendering is a well-known method in computer graphics to achieve photo-realistic images. In this paper we show how conventional image-based rendering algorithms can be extended to visualize general relativistic effects in a restricted class of spacetimes. We propose a generalized aberration formula in order to treat the visualization of special and general relativistic effects on the same footing. In this way, image-based general relativistic rendering can be regarded as an extension of special relativistic rendering. As an example, we present snapshots from the viewpoint of an observer traveling at warp speed.

Keywords: general relativity, image-based rendering, scientific visualization, warp speed

1 INTRODUCTION

Albert Einstein's Theory of Relativity is among the most famous areas of physics today. But despite the popularity, its abstract mathematical foundation renders it hard to comprehend. People usually have an intuitive understanding of flat three-dimensional space or a curved two-dimensional space. However, one does not have a notion of the flat four-dimensional spacetime Special Relativity deals with, let alone the curved four-dimensional spacetimes General Relativity uses to describe the effects of gravitational fields. Spacetime diagrams are the most widely used means to display relativistic properties. While being an appropriate tool for scientists, they are hardly understandable without prior knowledge, and therefore not feasible in order to convey general relativistic properties to a broader public.

To stimulate intuition, we suggest images of everyday objects as they would be seen in an environment dominated by relativistic effects. They are targeted at the fields of popular science and edutainment in the first place, but can be a useful visualization tool for scientists as well.

Unlike traditional, geometry-based techniques, a photo-realistic, well-known environment follows most naturally when using an image-based rendering scheme. Furthermore, the delicate and time-consuming step of fine-grained geometric modeling can be avoided. In this paper, we introduce an image-based approach to general relativistic rendering as an extension of special relativistic rendering[Weisk00c]. In a restricted class of spacetimes, a generalized aberration formula can be formulated in order to treat the visualization of special and general relativistic effects on the same footing.

The paper is organized as follows. In the following section, a brief overview on previous work is presented. Section 3 is focused on the physical and mathematical background for general relativistic image-based rendering. Here, the generalized aberration formula is introduced and the relativistic transformation of the plenoptic function is presented. Section 4 describes the relativistic extensions that have to be introduced in conventional image-based rendering techniques. In Section 5, we present details on the implementation and results. The paper ends with a short conclusion and an outlook on future work.

2 PREVIOUS AND RELATED WORK

Most of the previous work in relativistic visualization focuses on geometry and color transformations induced by Special Relativity. Hsiung and Dunn[Hsiun89] extended a classical ray tracer to display geometric distortions as seen by a fast moving observer. Later implementations[Hsiun90a, Hsiun90b] take into account color changes due to the Doppler effect as well. The T-buffer[Hsiun90c] is an alternative approach based on common polygon rendering that is able to visualize special relativistic geometry effects in real-time. Weiskopf[Weisk00a] proposed texture-based relativistic rendering for visualizing the apparent geometry and illumination of fast moving objects.

In computer graphics, the demand for photo-realistic image generation gave rise to image-based rendering (IBR) as a new, non-geometry-based rendering scheme. IBR today stands as a standard technique in computer graphics, QuickTime VR[Chen95] being one if its most well-known applications. More advanced techniques include plenoptic modeling[McMil95], light fields[Levoy96], the lumigraph[Gortl96], and view morphing[Seitz96].

IBR derives from the notion of the plenoptic function[Adels91] containing all visually perceptible information for each given point in spacetime. The plenoptic function allows to define more general cameras than the pin-hole camera commonly used in ray tracing applications. An exhaustive treatment of extended camera paradigms was given by Löffelmann and Gröller[Löffe96].

Image-based algorithms were adapted to special relativistic visualization[Weisk00c] in order to produce photo-realistic pictures including all special relativistic effects. In this paper, the image-based approach to special relativistic rendering is extended to more complex scenarios of General Relativity.

Many textbooks give a comprehensive introduction to General Relativity, the works of Weinberg[Weinb72] and Misner et al.[Misne73] being among the most popular and widely used. Previous work in general relativistic visualization is entirely geometry-based. Most implementations provide a proprietary general relativistic ray tracing system and confine themselves to a few simple setups with well-known metrics, like neutron stars and black holes[Ertl89, Nolle89, Nemir93, Nolle96]. A more general approach to non-linear ray tracing as a visualization technique was presented by Gröller[Gröll95]. Weiskopf[Weisk00b] investigated four-dimensional non-linear ray tracing in further detail and showed its applicability as a visualization tool in gravitational physics.

3 PHYSICAL BACKGROUND

The plenoptic function $P(x^\mu, \theta, \phi, \lambda)$ is a physical property defined as spectral intensity in a range of wavelengths between λ and $\lambda + d\lambda$ at a point x^μ in spacetime with the incoming light originating from the direction (θ, ϕ), given in spherical coordinates. P contains all information that is necessary to reconstruct the visual perception of an observer at a given point in spacetime.[1] It does not contain immediate depth information.[2]

For a fixed point in space, a discrete approximation of the plenoptic function can be composed from conventional images that are arranged into a spherical panorama. Samples of the plenoptic function can thus be taken by means of a calibrated camera as described, e.g., in [Weisk00c]. Relativistic visualization has to construct a transformation of the plenoptic function from our everyday world into arbitrary systems that exhibit the desired relativistic effects. For Special Relativity, this means that we record the plenoptic function in a frame of reference that is approximately at rest with regard to our sample objects, and later generate snapshots as would be seen by a fast moving observer. For general relativistic visualization, we record the plenoptic function in an approximately flat spacetime[3], and later transform into snapshots as would be seen if the surrounding gravitational forces were much stronger and no longer negligible.

In Special Relativity, light in vacuo travels along straight lines. The geometric properties of a light ray as a whole can thus be fully described by a point and a direction in space at this point. Therefore it is possible to state the transformation of a light ray between two special relativistic frames of reference as a pointwise transformation, the Lorentz-transformation of the plenoptic function[Weisk00c]. The relative speed of the two frames of reference is the only parameter in this transformation. Neglecting color transformations, special relativistic effects are contained in the aberration equation (figure 1(a)),

$$\theta' = \arccos \frac{\cos\theta - \beta}{1 - \beta\cos\theta}, \qquad (1)$$

where θ and θ' are the angles of an incident ray of light as measured in two inertial frames of reference S and S'. S' is moving with velocity $v = \beta c$ relative to

[1] Information on the polarization of the incoming light is usually neglected in the plenoptic function because the human eye is not sensitive to polarization.

[2] Though some three-dimensional information can be reconstructed from the plenoptic function at multiple points in space.

[3] Gravitational forces on earth are weak enough for the spacetime to be reasonably flat, i.e., light paths on earth can be assumed to be straight lines.

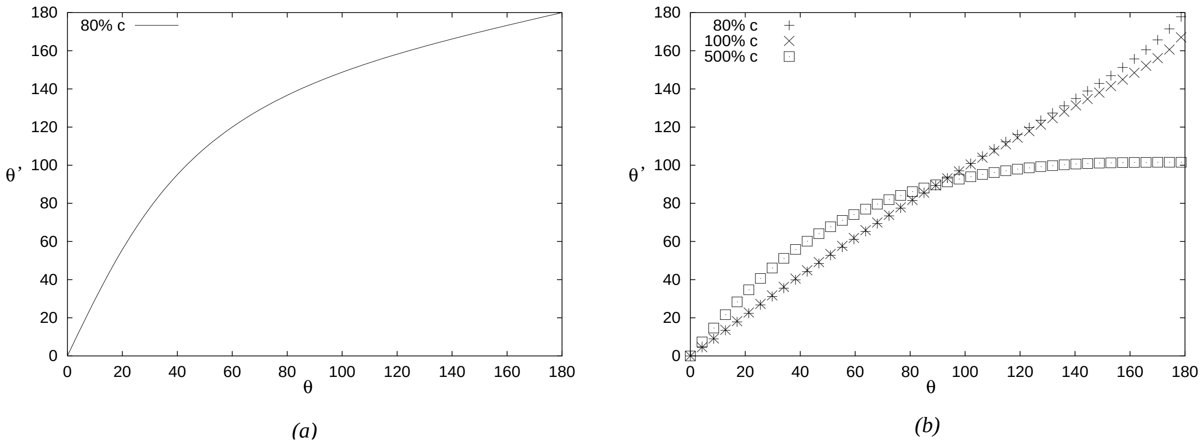

Figure 1: *(a) The special relativistic aberration function for a relative speed of* $0.8c$. *(b) The generalized aberration function for the warp metric at apparent speeds of* $0.8c$, $1c$, *and* $5c$. *Unlike their special relativistic counterpart, all curves pass through* $(90°, 90°)$. *This means that front view and back view in the warp metric stay clearly separated, whereas in special relativity at high velocities, objects to the back become visible.* θ *and* θ' *are given in degrees. Similar diagrams are presented in [Clark99].*

S; c is the speed of light. Both angles are taken with respect to the direction of the relative motion of S'.

General Relativity introduces a curved four-dimensional spacetime to take into account gravitational effects. Differential geometry presents the mathematical foundation of General Relativity. Its most fundamental property is the metric tensor—or metric for short—, that contains all information about the curvature of spacetime, or more physically speaking, of the gravitational forces. In the absence of a gravitating mass, the metric is identical to the Minkowski metric known from Special Relativity, the corresponding spacetime is called to be flat. In General Relativity, a gravitating mass gives rise to non-trivial terms in the metric tensor. Accordingly, the spacetime is curved. Due to the curvature, light rays no longer follow simple straight lines. Instead, light travels along so-called null geodesics, given by the geodesic equation

$$\frac{d^2 x^\mu(\alpha)}{d\alpha^2} + \sum_{\nu,\rho=0}^{3} \Gamma^\mu{}_{\nu\rho}(\vec{x}) \frac{dx^\nu(\alpha)}{d\alpha} \frac{dx^\rho(\alpha)}{d\alpha} = 0, \quad (2)$$

and the null condition, confining the solutions of (2) to lightlike paths,

$$g_{\mu\nu}(\vec{x}) \frac{dx^\mu}{d\alpha} \frac{dx^\nu}{d\alpha} = 0. \quad (3)$$

$g_{\mu\nu}$ is the metric tensor at position $\vec{x}$ in spacetime, x^μ denotes the associated coordinates of the light ray. The $\Gamma^\mu{}_{\nu\rho}$ are the so-called Christoffel symbols that can be calculated from the metric and its first order derivatives. α is used to parameterize the path. Greek indices run from zero to three. Given a starting location and direction in spacetime, the geodesic equation yields an initial value problem for a system of non-linear ordinary differential equations. It is known from the theory of differential equations that there exists a unique solution to this problem, rendering a unique path of a light ray.

Because of the complex nature of light paths in General Relativity, a generic approach to render relativistic images needs to take into account the full four-dimensional layout of a scene. Under certain conditions, however, it is possible to maintain an aberration-like view as in the special relativistic case, which makes it feasible to use image-based rendering techniques: First, the introduced spacetime needs to be asymptotically flat. Second, all points in the curved region of spacetime must be closer to the observer than any visible scene object[4]. These two constraints allow a sphere to be drawn, centered at the observer's location, that separates space into a general relativistic part inside, and an entirely non-relativistic part outside (figure 2). As all light originates from objects outside, the inbound part of the plenoptic function on the sphere's surface is independent of the particular shape of spacetime inside. In order to obtain general relativistic images, it is therefore sufficient to know the plenoptic function on the sphere's surface, and the behavior of the light rays within this sphere.

The full plenoptic function is usually hard to record, so for practical purposes, further restrictions have to be imposed. First, if all scene objects are far away from the observer compared to the radius of the sphere, the plenoptic function at all points on

[4] There have to be invisible objects as well in order to build up the curved region in spacetime. Invisibility may be inherent to the objects like, e. g. gravitational waves, due to their optical or spacial properties like, e. g., sparsely distributed matter, or as simple as the object lying outside the observer's field of view.

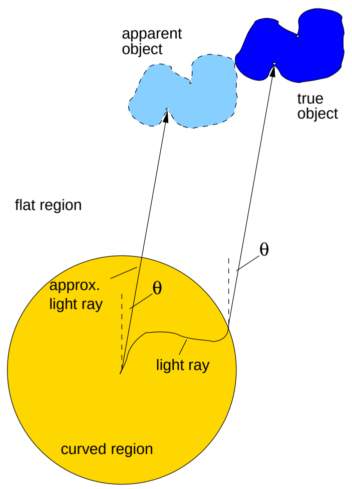

Figure 2: *If gravitational forces are confined to a limited region in space, a sphere can be imagined around the observer that separates curved and flat regions in spacetime. Light in the flat region travels along straight lines. If the sphere's radius is small compared to the distance to the object, the plenoptic function at the surface can be approximated by the plenoptic function at the sphere's center. For nearby objects, this approximation introduces parallax artifacts.*

the sphere's surface may be approximated by the plenoptic function at the center of the sphere in the flat source spacetime. Second, in a static scene, the plenoptic function on the sphere's surface is independent of time. Combining both restrictions, a single 4π sterad view is sufficient to describe the plenoptic function at all points on the sphere. Note that the metric may still be time-dependent. Third, for a cylindrically symmetric spacetime, spherical coordinates may be defined so that the metric is independent of angle ϕ. If all these prerequisites are met, an aberration equation similar to (1) can be formulated to describe the connection between a flat spacetime and a curved spacetime with strong gravitational forces,

$$\theta' = f(\theta). \tag{4}$$

The generalized aberration function f depends only on the observer's location and on the metric inside the sphere. In this equation, possible absorption or optical diffraction by the matter building up the metric is neglected.

As a sample application, we investigate the so-called warp metric[Alcub94], a physically sound solution to Einstein's general relativistic field equations that allows a body to travel faster than light. The body itself rests inside a warp bubble; it is the bubble that moves through space and carries the body with it. For an outside observer, the body appears to move at warp speed—faster than light.[5] The warp metric is time-dependent but meets all restrictions stated above. It is cylindrically symmetric and differs from flat spacetime only in a small region at the bubble's surface. Assuming a point-like observer, the bubble can be constructed infinitely small around the observer's location; in this sense, all scene objects are guaranteed to be far away compared to the bubble's extent.

The warp metric was alternatively visualized by means of four-dimensional ray tracing in [Weisk00b]. Clark et al.[Clark99] investigated null-geodesics in the warp metric on a physical footing.

Further well-known metrics include the Schwarzschild metric for static, spherically symmetric bodies, and the Kerr metric that takes into account an additional rotation of the body. Both metrics show the required cylindrical symmetry but are unlimited in spacial extent. However, they are asymptotically flat, so given a certain degree of accuracy, a cut-off radius can be defined and the metric be regarded as flat on the outside. In this way, the proposed rendering scheme can be applied to visualize the looks of faraway objects—like distant stars—as seen by an observer close to a static gravitating mass, or an observer located on the symmetry axis close to a rotating gravitating mass.

4 RENDERING TECHNIQUE

Aberration-based relativistic rendering is a straightforward extension to the rendering pipeline in traditional image-based rendering. It introduces an additional transformation of the plenoptic function used for final image generation. This transformation consists of conventional three-dimensional rotations to orient the metric in space, and the calculation of aberration according to (4). Shifts in wavelength and intensity are currently neglected. The relativistic modifications are located at the end of the rendering pipeline, just before final image generation; all prior steps are left unchanged.

This rendering scheme can be regarded as a subset of the extended camera model by Löffelmann and Gröller[Löffe96] that was originally developed for ray tracers. It can, however, trivially be applied to imaged-based renderers as well. We confine our extended camera to a fixed point in spacetime. No such prerequisites are imposed on the directional mappings that are given by the generalized aberration function.

[5] While the warp metric does not violate the Theory of General Relativity, constructing a warp bubble requires so-called exotic matter with negative rest energy[Ford00]. Exotic matter still is in accordance to General Relativity but has never been observed so far. It may or may not exist.

(a)

(b)

Figure 3: *Snapshot of a sample scene at rest. (a) Front view. The camera points into the direction that will later be used as direction of flight. It covers a horizontal angle of view of 60°. This camera parameter remains fixed for all further images. (b) Back view.*

In this sense, image-based relativistic visualization can be regarded as viewing a real-world scene through an extended camera with a generic relativistic lens applied.

By capturing a series of plenoptic functions along a predefined path, a sequence of relativistic snapshots can be generated and combined into a relativistic animation. In regions where areas of the source images are heavily scaled down by the relativistic transformation, numerical errors due to the discrete nature of the input data may lead to disturbing flickering artifacts in parts of the movie. This problem is known in traditional computer graphics as well and can be addressed by filtering techniques. We treat filtering issues in detail in the following section.

5 IMPLEMENTATION AND RESULTS

The described image-based relativistic rendering scheme is implemented in a batch-job oriented software renderer called *Ergänzingen*. It extends the software system described in [Weisk00c]. The renderer is written in C++. It takes a series of images captured at a single point in space, and blends and stitches them into a 4π sterad view, a spherical panorama. The generalized aberration function (4) is obtained numerically: the initial value problem for the geodesic equation (2) is integrated by means of the fourth-order Runge-Kutta method[Press94] with adaptive step-size control. For symmetric metrics such as the warp metric, this is done in a pre-computing step for a discrete set of θ samples and stored in a lookup table. Intermediate values are later obtained by linear interpolation.

Aliasing effects are reduced by means of bilinear interpolation on the source images. Alternatively the source image can be thought of as a two-dimensional texture with regard to coordinates θ and ϕ, so texture filtering techniques can be applied. MIP mapping[Willi83] as the most widely used filtering technique is based on quadratic footprints, while the highly non-linear relativistic transformations cause irregularly shaped footprints. We have therefore implemented a filtering scheme that calculates a rectangular axis-aligned footprint of each source pixel based on the first-order derivatives of the transformation functions. Additionally, standard supersampling can be applied. *Ergänzingen* so far only visualizes apparent geometry.

Raw image data for the supplied sample images in figures 3–7 was captured using a standard DV camera mounted on a telescope fork arm. The camera is calibrated on the fork arm to ensure that its optical center remains fixed in space when the camera is turned to different positions. So multiple views from a single point are obtained that can be combined into a full 4π sterad panorama. Positioning and image capturing are automated and remotely controlled by a laptop.

The following example investigates a real-world scene (figure 3) at high speeds. Figures 6 shows a series of front view images rendered from the center of a warp bubble traveling at various speeds. Plots of some corresponding generalized aberration functions are displayed in figure 1(b). Notably, while the apparent field of view gets enlarged in the warp-drive front view, straight lines remain straight, and angular distortions are weak. As the aberration transformation depends only on angle θ, this is by no means natural but a very special case, especially when compared to the special relativistic results (figure 4), where straight lines become distorted to hyperbolae. Note also that this property is immediately visible from the rendered images, yet hardly apparent from the data plots.

(a) (b)

Figure 4: *Comparing the front view at eighty percent of the speed of light. (a) Special relativistic view. (b) View from inside the warp bubble. Image distortions inside the warp bubble are remarkably small compared to the special relativistic result. At warp speed straight lines remain straight in the front view, while special relativistic effects distort them to hyperbolae.*

(a) (b)

Figure 5: *Comparing the back view at eighty percent of the speed of light. (a) Special relativistic view. (b) View from inside the warp bubble. While Special Relativity magnifies objects to the back, the warp bubble shows the opposite effect, and the apparent field of view gets even larger than at rest.*

Looking opposite to the direction of motion offers a slightly distorted view at velocities well below the speed of light (figure 5). At warp speed, light from a cone shaped region in space cannot reach the observer anymore (figure 7). The properties of the aberration function however ensure that there is no apparent black void; instead, the virtual hole gets sewn up with image information from the surrounding areas.

6 CONCLUSION AND FUTURE WORK

In this paper we have shown how to extend image-based methods from special relativistic visualization to render general relativistic scenes. An extended aberration function has been described which allows to treat the visualization of special and general relativistic effects on the same footing. We have presented an analysis of the requirements metric and scene must meet for the method being applicable. Photo-realistic snapshots of objects as seen through strong gravitational fields can be generated at ease. These images provide additional insight in the properties of certain spacetimes and are applicable for scientific visualization as well as edutainment.

Further work in this area will improve the geodesic calculator to include information about the gravitational blue- or redshift, so relativistic effects on color and intensity can be taken into account as well. Advanced texture-filtering techniques more suitable for irregularly shaped footprints will be investigated in order to enhance image quality. An improved automated system to capture multiple panoramic images along a pre-defined path will allow to generate photo-realistic general relativistic movies from a series of snapshots.

(a) (b)

Figure 6: *Front view at warp speed. (a) 500 percent of the speed of light. (b) 1000 percent of the speed of light. An apparently wider field of view is the only prominent effect.*

(a) (b)

Figure 7: *Back view at warp speed. (a) 100 percent of the speed of light. (b) 120 percent of the speed of light. Geometric distortions are much more prominent than in the front view (figure 6). At higher speeds, image information from a cone shaped region to the back is no longer visible.*

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