SIMPLIFIED PROCEDURES FOR MODELLING FREE-FORM SURFACES

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ABSTRACT

Polynomial representations, such as Bezier curves, B-splines or NURBS, form the basis of numerous surface modelling packages. Although they possess properties that result in fast, efficient software, they have some limitations in terms of the shapes that can be modelled and the nature of the controls they offer the user. This paper presents a new, point-based approach to modelling that is guaranteed to produce high quality surfaces whilst providing a simpler, more intuitive user interface. These qualities make the method ideal for such applications as graphical design or animation where there is a need to produce pleasing images with minimum time and effort.

Key words: geometric modelling, computer graphics, geometric control, curvature continuous

1 INTRODUCTION

The use of polynomials to represent shape has played a long and successful part in the development of surface modelling. The combination of stable, efficient algorithms and highly developed software has ensured that polynomial-based methods have become the norm. It is therefore tempting to forget that this approach has a number of limitations that can seriously inconvenience the user.

The fundamental problem is that polynomial representations can only provide optimal results when the object to be modelled is polynomial in shape or nearly so. If other shapes are attempted, it may be difficult to prevent the formation of ripples or to obtain smooth blends between the patches that define the surface model, see [Ball95], [Ball96], [Davis63]. The user's task is complicated by the non-geometric nature of the controls provided: that is, control polygons comprising points lying off the surface or sets of weights. The need to decompose a complex object into a mesh of simpler patches, and to do so efficiently, also places demands on the user's knowledge and experience.

In this paper, we describe a fundamentally new approach to surface modelling that is designed to avoid these pitfalls. Instead of using polynomials to represent shape, we return to the object's geometry and fit a surface through a user-selected grid of points that lie on it. The fitting procedure guarantees the quality of the surface and permits localised shape refinement by adjustment of the grid. The method does not require partition of the grid and so the twin problems of surface sub-division and patch blending do not arise. These features provide the user with a simple and effective interface.

The point-based method provides the freedom to model *ad hoc* shapes. The surface fitting procedure is based on segments with a linear variation of curvature and uses algorithms that ensure positional and tangential continuity. Taken together, these properties guarantee a fairness of form and fit, irrespective of the underlying shape.

2 MODELLING SURFACES

2.1 General Overview

This section introduces the structure of the point-based modelling technique and outlines its mathematical basis. Details of the mathematics and algorithms are deferred to later sections.

The method addresses the problem originally considered by Ferguson [Fergu93] who also sought to fit a surface through a grid of points. The fundamental assumptions under-pinning the present method is that the grid is sufficiently dense to characterise the required shape and that a piecewise curvature continuous surface can approximate the shape to within the given tolerance.

The method rests upon the Generalised Cornu Spiral (GCS), a planar segment with a rational linear curvature variation, as a means of interpolating between pairs of points. The curve fitting algorithm exactly matches positions and tangents of both end points, these data being estimated by circular interpolation of neighbouring grid points. The GCS algorithm is used to fit curves in the planes that pass through both end points and contain each of the tangents. A stable, well-defined procedure is used to construct a three-dimensional space curve from the two projections. It is then a simple matter to interpolate for positions, tangents and curvatures within the curve.

By sweeping the interpolation procedure across the rows and columns, it is possible to increase the density of the grid by inserting new points. This enables geometrical data to be calculated at any location within the surface so that high quality graphics can be produced.

A significant benefit of the new approach is that the GCS-based interpolation procedure guarantees the generation of high quality curves and surfaces without the need for user intervention. It is well known that the curvature profile of a planar curve determines its shape to within a solid body movement. See [Nutbo88] for example. It follows that a well behaved curvature profile will always result in a smooth curve that is free from ripples or other undesired phenomena.

Since curvature is a second derivative quantity, the method also ensures tangential continuity between neighbouring segments, even if curvature discontinuities are present at the join.

We observe that cubic splines exhibit the inverse of these properties: although curvature continuity is assured between segments, there is no curvature control within them and so user manipulation may be required if pleasing curves are to be obtained. Ali et al [Ali99] provide further discussion of this point.

2.2 Input Data

It is assumed that the grid of points is sufficiently dense to characterise the desired shape. This does not imply that a high density 'data cloud', characteristic of many reverse engineering systems, [Hosch96], is required to drive the method. Case studies on a wide variety of shapes, ranging from motor panels to a shoe, have shown that the data requirement and accuracy of the present procedure is closely comparable with that of Bezier curves or NURBS.

In order to ensure high accuracy, it is assumed that points are sampled using a spacing that is regular and geometric in nature. Surfaces that contain significant features are modelled most accurately when the grid runs parallel or perpendicular to those features. This is consistent with normal practice when defining or digitising sculptured shapes and is therefore not considered to be a significant limitation of the method. As a matter of convenience, the following discussion is focused on a grid with rectangular topology but this is not a fundamental restriction.

2.3 Estimating Tangents and Curvatures

Tangents and curvatures at grid points may be estimated using a technique based on circular interpolation. Consider a string of five consecutive points $\mathbf{p}_{i-2}, ..., \mathbf{p}_{i+2}$ that lie within the grid. If the curvature vectors at $\mathbf{p}_i$ of the interpolating circles through $\{\mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}\}$ and $\{\mathbf{p}_{i-2}, \mathbf{p}_i, \mathbf{p}_{i+2}\}$ are denoted by $\mathbf{k}_i^1$ and $\mathbf{k}_i^2$ respectively, then the Richardson extrapolation procedure [Boehm93], may be used to obtain an improved estimate for $\mathbf{k}_i$:

$$\mathbf{k}_i = \frac{(s_{i-2}+s_{i-1})(s_i+s_{i+1})\mathbf{k}_i^1 - s_{i-1}s_i\mathbf{k}_i^2}{(s_{i-2}+s_{i-1})(s_i+s_{i+1}) - s_{i-1}s_i}$$

where s_i denotes the arc length between grid points $\mathbf{p}_{i-1}$ and $\mathbf{p}_i$. It can be shown [Tooke97a] that $\mathbf{k}_i$ is a second order approximation to the principal curvature vector when the data is approximately regularly spaced. The approximation may still be used even if the data is irregularly spaced because the procedure is well-defined and continuous for any combination of arc lengths. It is also invariant with respect to constant scaling and reversal of point ordering.

The arc lengths used in the previous equation may also be estimated using circular interpolation. Two estimates of s_i, the length between $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$, may be obtained by fitting circles through the four points $\{\mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}, \mathbf{p}_{i+2}\}$, the final estimate being taken as their arithmetic mean to remove any bias.

Similar procedures may be used to provide second order estimates of the unit tangent at each grid point. Details, together with error estimates and sample calculations may be found in the paper by Tookey and Ball [Tooke97a].

2.4 Interpolation of Planar Curves

As outlined in section 2.1, the key element of the point-based modelling system is a three-dimensional space curve that spans a pair of grid points. This curve is constructed from two planar projections, one in each of the planes that pass through both end points and contain one the unit tangents. The algorithm to compute these curves is outlined below.

Ali [Ali94] defines a Generalised Cornu Spiral (GCS) to be a planar curve along which the curvature, κ, varies as a rational linear function of the arc length, s :

$$\kappa(s) = \frac{(\kappa_1 - \kappa_0 + r\,\kappa_1)\,s + \kappa_0 S}{rs + S} \qquad (1)$$

where subscripts 0 and 1 denote the curve's end points and S is the total arc length. The initial estimates of the end point curvatures and r, a shape factor, are adjusted under iteration to ensure that the curve exactly fits the tangents at the end points. If φ is the total winding angle between the end points, then Ali shows that r may be computed from:

$$\varphi = \frac{S}{r^2}\left\{(1+r)\,(\kappa_0-\kappa_1)\,\log(1+r) + r\left[(1+r)\kappa_1-\kappa_0\right]\right\} \qquad (2)$$

The position of end point 1 of the GCS may be computed using a numerical procedure to evaluate the Fresnel integrals [Nutbo88] :

$$x_1 = x_0 + \int_0^S \cos\left[\int_0^s \kappa(t)\,dt\right] ds \qquad (3a)$$

$$y_1 = y_0 + \int_0^S \sin\left[\int_0^s \kappa(t)\,dt\right] ds \qquad (3b)$$

Using an initial estimate of the total arc length, S', equations (2) and (3) may be placed under an iteration scheme that adjusts the end point curvatures, κ_0 and κ_1, until a curve that subtends the correct chord angle is obtained, see [Ali99]. The method of false positions is suitably stable and efficient for this purpose, see [Boehm93].

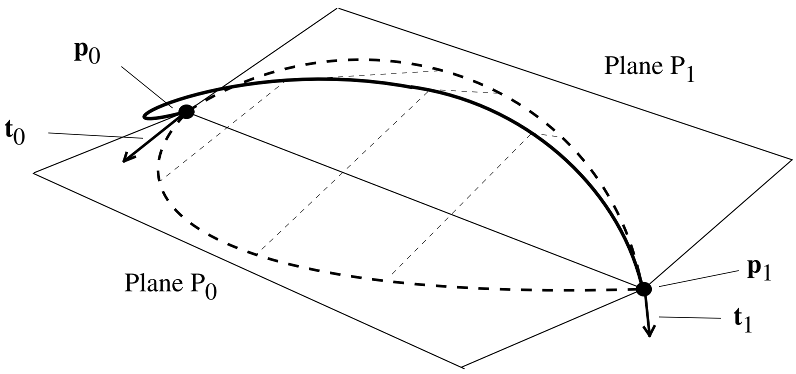
Although the resulting curve matches tangents correctly, it must be scaled if the computed position of end point 1 is to coincide with the given position. If primed variables denote values produced by the iteration scheme and un-primed variables denote the required values, then it can be shown, [Ali94], that the appropriate scalings are:

$$S = \rho\,S' \qquad k_0 = \kappa'_0/\rho \qquad k_1 = \kappa'_1/\rho$$

where the scale factor, ρ, is given by:

$$\rho = \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_1'^2 + y_1'^2}}$$

Once these values have been determined, positions, tangents and curvatures may be calculated at any point within the segment.



Construction of Space Curve from Projections
Figure 1

2.5 Construction of Space Curves

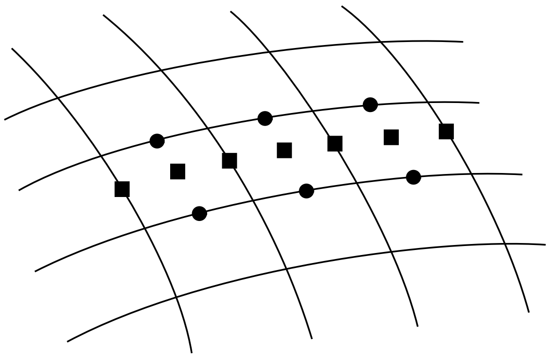
Fig. 1 illustrates the method used to construct a space curve from its planar projections. Plane P_0 passes through both end points and contains the unit tangent at end point 0. The curvature and unit tangent at end point 1 may be projected onto P_0, thus providing all the data needed to define the GCS that lies in it. Similar computations may be performed for the GCS lying in plane P_1, the latter being aligned with the unit tangent at end point 1.

The curve construction procedure has been shown to be well-defined and stable. Again, expressions are available to compute positions, tangents and curvatures at any location within the curve, the relevant formulae being given in Ali's thesis [Ali94] or Tookey’s paper [Tooke97b].

2.6 Interpolation within Surfaces

The space curve interpolation procedure may be used to compute geometrical data at any location within the surface defined by the grid. See Fig. 2 where it has been applied to each row of points to compute the ‘half points’ that lie mid-way along each segment. These are marked by circles. The procedure may then repeated, but this time sweeping along columns of points, including the newly calculated ones, to complete the grid of half points. The points produced during the second sweep are marked by squares.

To eliminate any bias, the sweeping procedure is applied twice: once in “first row then column” order and then in the reverse order. The arithmetic mean of the two sets of estimates is then used to enlarge the grid.



Increasing the Density of the Grid
Figure 2

This process may be applied recursively to generate the limit set of the object to within a specified tolerance. Experiment has shown the point insertion process to be stable and well behaved. Case studies, involving comparisons with polynomial-based methods, have shown that the process to be of similar accuracy as conventional CAD systems.

3 AUTOMATIC SURFACE REFINEMENT

Simple, intuitive controls for the refinement of the surface model are a key feature of the point-based method. Shape may be adjusted by moving, inserting or deleting points, safe in the knowledge that the fitting procedure will produce a smooth surface that matches positions and tangents exactly and does not contain extraneous ripples between grid points.

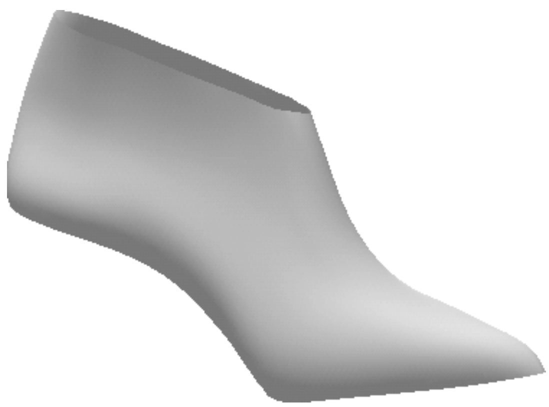
This idea has been carried a step forward to produce automatic procedures that simulate the sanding and filling operations used by craftsmen when producing a finely finished artefact. Diagnostic tools have been developed for the assessment of surface quality, including contour plots of arc length, tangent and curvature. For example, see [Cripp98]. These make it possible to identify regions where the surface contains flaws due to errors in the input data. The interpolation technique may then be used to adjust the positions of such points so that they lie in a surface that is compatible with the rest of the data. Again, the curvature control imposed by the GCS ensures that the amended surface will be of high quality.

4 CASE STUDY

Figures 3 and 4 illustrate the application of the point-based method to modelling a shoe. The data was kindly supplied by C & J Clark International, an industrial collaborator.

Data Used to Define the Shoe
Figure 3

Fig. 3 shows the input data, which was generated by the company’s non-rational bisextic Bezier-based modelling package, and Fig. 4 shows a rendered image generated from it.



Rendered Image of the Shoe
Figure 4

Fig. 4 was produced by a double application of the surface interpolation technique, so as to increase the density of the grid by a factor of four. The new points were triangulated and then positions and tangents were fed into OpenGL to obtain the final image.

This and other case studies have shown that the present method regularly produces surfaces that are within 0.01mm of those produced by Bezier or NURBS-based software. It is interesting to note that larger discrepancies arose only in those regions where users had difficulty modelling shape using polynomial-based software.

5 CONCLUSIONS

We have presented a point-based approach to the modelling of free-form surfaces. The approach has significant benefits in terms of: i) its ability to model polynomial and non-polynomial shapes; ii) its simple but powerful user interface and; iii) the provision of facilities that automatically 'tidy up' noisy input data. The method is based on segments with a linear variation of curvature and this ensures the quality of the resulting surface. Case studies have shown that the data requirement and accuracy are comparable with current CAD technology.

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7 REFERENCES

[Ali94] Ali, J: Geometric Control of Planar Curves, PhD thesis, The University of Birmingham, UK, 1994.

[Ali 99] Ali, J M, Tookey, R M, Ball, J V, Ball, A A: The Generalised Cornu Spiral and its Application to Span Generation, *Journal of Computational and Applied Mathematics*, Vol. 102, pp. 37-47, 1999.

[Ball95] Ball, A A: Geometry Based Surface Modelling. Proceedings of 11th International Conference on Computer Aided Production Engineering, IMechE, London, pp. 49-54, 1995.

[Ball96] Ball, A A: Simple CADCM Procedures for Complex Shaped Components, International Journal of Production Research, Vol. 34, No. 9, pp. 2461-2469, 1996.

[Boehm93] Boehm, W, Prautzsch, H: Numerical Methods, A. K. Peters, 1993.

[Cripp98] Cripps, R J, Howe, R E: Surface Visualisation and Assessment Using Geometric Parameter Curves. Proceedings of 14th NCMR Conference, The University of Derby, UK, 1998.

[Davis63] Davis, P J: Interpolation and Approximation, Blaisdell Publishing Company, 1963.

[Fergu93] Ferguson, J C: F-Methods for Free-Form Curve and Hypersurface Definition, in Fundamental Developments of Computer-Aided Geometric Modelling, Editor L. Piegl, Academic Press, pp. 99-115, 1993.

[Hosch96] Hoschek, J, Dankwort, W: Reverse Engineering, B G Teubner, 1996.

[Nutbo88] Nutbourne, A W, Martin, R R: Differential Geometry Applied to Curve and Surface Design, Volume 1: Foundations, Ellis Horwood Ltd., Chichester, 1988.

[Tooke97a] Tookey, R M, Ball, A A: Estimation of Curvatures from Planar Point Data, in The Mathematics of Surfaces VII, Editors T. Goodman and R. Martin, Information Geometers, pp. 131-144, 1997

[Tooke97b] Tookey, R M: Interpolatory Subdivision and Bounding Box Construction for Grids of Points. Internal Report 97/03, Geometric Modelling Group, School of Manufacturing and Mechanical Engineering, The University of Birmingham, UK, 1997.