CONSTRAINED SURFACE MODELING AND DEFORMATION

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ABSTRACT :

Soft surface modeling allows creation or reconstruction of various objects. We propose a deformable surface model, relying upon operations on local and global surface properties. This model includes an initial shape, various constraints on the surface and external operators. Besides, surface hierarchy confers local properties that allows the easy satisfaction of various constraints. The originality of our work includes the addition of patterns in surface representation, the use of an alphabet to represent the resulting hierarchical surface and new 3D development tools such as the "profiler".

Keywords : surface modeling, deformation, minimal energy under constraints, B-Splines, refinement.

1 INTRODUCTION

Our goal is to model objects with interactive and intuitive tools, directly acting on surfaces (points on a surface, profiler tool, …). Also should we be able to reconstruct any object for which the global shape is known a priori, and which is deformed by real data (such as echographic images in the case of hip for instance).

Elements that guide surface modeling may be of various kind. They may describe a priori knowledge on the surface shape, or locally describe the surface (point, curve, surface patch included in the surface to model), or influence the shape (attracting or repulsion point, part of surface profiler). All these elements have local influence, except knowledge of the surface shape which has global influence.

Locality of the deformation process is important. In the modeling case, we may add local details without altering the global shape. In the reconstruction case, a lack of real data has to be replaced by new local elements while keeping the characteristics of the initial surface for the deformed surface.

Model deformation is driven by an energy minimization under constraints, composed of two terms. An internal energy that rules global deformation and an external energy that characterises local deformations. First, we present our surface model and its originality compared to previous works. Then we explain the model deformation including new 3D tools and energy minimization.

2 SURFACE MODEL

2.1 Choice of a model

Our goal is to give a smooth and malleable surface locally and interactively deformable. Numerous works have been done. [Seder86a], [Coqui90a] proposed FFD (Free Form Deformation) that embeds an object in a 3D lattice and deforms it by acting on the lattice vertices. A drawback of FFD is that many control points have to be manipulated in order to solve surface constraints. [Hsu92a] tried to solve this problem by a direct manipulation of FFD. Other reconstruction and modeling methods use dynamics together with oriented particles [Szeli92a] or implicit surfaces [Blinn82a], [Gascu93a]. These methods imply many basic objects (particles, blobs,…) and the solving of motion equations with delicate numerical integration.

[Terzo87a], [Terzo94a] proposed an elastic deformable model including dynamics in a geometric model. This approach has allowed many developments in surface modeling and reconstruction. For instance, [Celni91a] deforms surface elements in order to design free form shapes; [Welch92a] builds B-Splines surfaces with constraints.

A priori knowledge of the object shape to model or reconstruction yields us to define an initial shape [Terzo87a], that we deform to satisfy global and local constraints.

The use of external operators allows giving a local shape to the surface. This yields to add an external energy to the internal energy characterizing the surface shape [Terzo87a], [Veltk95a], [Grein94a], [Grein96a].

2.2 Surface representation

Our aim is to get a surface with different levels of detail in order to solve local deformation problems. B-Spline surfaces are commonly used surfaces for such problems, because they are smooth with local properties and easily refinable by increasing the number of control points.

As we shall see in §3, our problem is formulated as an energy system to minimize, in which the unknowns are the control points. We must ensure enough degrees of freedom for this system. If two B-Spline surface patches are adjacent, a p-band of control points is used to ensure C^{p-1} continuity. For instance, in a bicubic patch, up to p^2-1 control points may be submitted to continuity constraints. This is the reason why we define a "pattern" as a piece of surface (build from a set of patches) for which the only boundaries are concerned by continuity constraints. This yields to significatively increase the number of degrees of freedom in a refinement process.

A non-uniform surface is then the sum of uniform patterns of varying levels (like a patchwork). The object surface is composed of identical patterns but with varying refinement levels.

Compared to [Welch92a], we increase the surface local behaviour by making each pattern independent of its refinement level. In fact, in our model, if a pattern consist of n control points, its refinement will add 4*n points to number of control points (the system size).

In [Welch92a], two cases arise : either the refinement level has already been created and no control points are added or it has not been created and the system size is increased of the total number of control points of this refinement level.
Besides, in our model, the creation of a new pattern by a refinement process is independent of other patterns (patterns do not share control points). For a unique local refinement, we do not need to increase the number of control points for the whole surface. For example in the case of a bicubic B-Spline, starting with 4 patches that need 25 control points, after two refinements Welch’s model needs 25+49+81=155 control points (Fig. 1 upper line). In our model we only have 25+25+25=75 control points (Fig. 1 lower line).

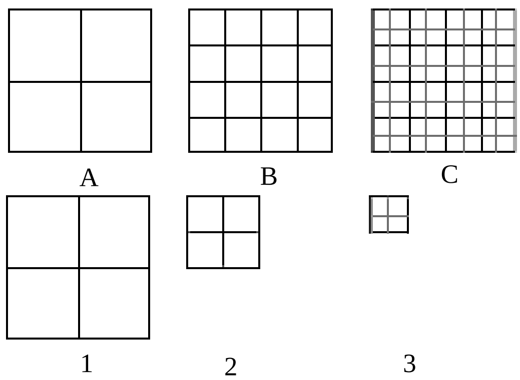

Fig. 1 A (level 0), B (level 1), C(level 2) compose the control points vector of Welch’s model, and 1 (pattern 0), 2(pattern 1), 3 (pattern 2) compose the control points vector of our model

The object surface appears to be a sum of B-Splines uniform patches that compose a pattern. A B-Spline patch is [Barte87a]:

$$Q_{ij}(u,v)=\sum_{r=-p}^{0}\sum_{s=-q}^{0}V_{i+r,j+s}\,b_r(u)\,b_s(v) \qquad (1)$$

Such that the i,j[th] patch is completly determined by (p+1)*(q+1) control points and where b(t) is the vector of basis functions evaluated at t. The degree of the surface patch is p and C^{p-1} continuity between two patches is ensured by sharing a p-band of control points.

A pattern is defined as a set of neighbouring B-Splines patches (they share control points):

$$M_k(u,v)=\sum_i^r\sum_j^s V_{i,j}b_i(u)b_j(v) \quad (2)$$

Where M is a uniform B-Spline surface composed of (r+1-p)*(s+1-q) surface patches. By converting the $V_{i,j}$ matrix in a V_k vector [Welch92a], Eq. 2 can be re-written

$$M_k(u,v)=V_k.b_k(u,v) \quad (3)$$

Where $b_k(u,v)$ is the vector of basis functions and V_k is the vector of control points. The whole non-uniform surface is the sum of patterns (see Fig. 2).

$$S(u,v)=\sum M_k(u,v) \quad (4)$$

or

$$S(u,v)=V^T b(u,v) \quad (5)$$

Where V and b are the concatenation on the whole parametric domain of the patterns' control points and of the basis functions respectively.

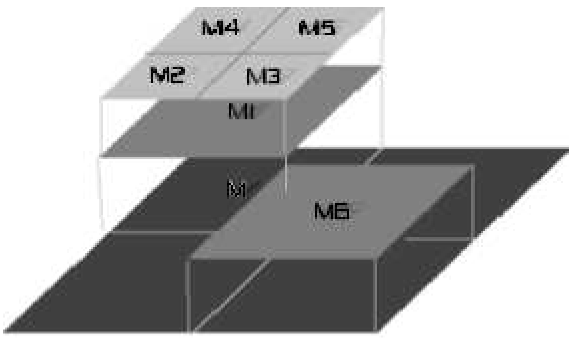

Fig. 2 Non-uniform surface representation as sum of patterns (M,M1,…,M6)

Continuity between patterns is ensured. Although they are independent (ie each is defined by its own control points), the surface is kept continuous by following two rules :

-rule 1: if two neighbouring patterns are of different levels, we constrain a band of control points associated with each pattern's boundary to be 0. This band width varies according to the degree of continuity desired.

-rule 2 : if two neighbouring patterns are of same level, we constrain the bands of control points associated with each pattern's boundary to be equal. Again, the band width varies with the degree of continuity.

This second rule allows to refine a pattern independently from the other patterns of the same refinement level.

The more complex the shape of the object, the more the pattern is refined. Parametric domain subdivision is performed by a classical Oslo algorithm [Forse88a], [Barte87a]. The choice of a pattern for the surface depends on the complexity of the shapes (particularly of levels of details). To solve numerous constraints, the energy system needs many degrees of freedom. A pattern constituted of few patches and submitted to many constraints yields to the failure of the energy system minimization which involves the iterative necessary subdivisions of this pattern. A better choice may be to start with a pattern constituted of more numerous patches.

3 SURFACE DEFORMATION

3.1 Deformation internal energy

Previously, we have seen how to represent the surface. Now we must control it. A surface is ruled by its internal energy. Internal energy of a deformation from an initial surface S_0 to a deformed surface S is [Terzo87a]:

$$E_{\text{int}}(s)=\int_s \left\|G-G^0\right\|_\alpha^2+\left\|B-B^0\right\|_\beta^2 ds \quad (6)$$

in which G et B are the first and second fundamental forms of the deformed surface S. G is related to stretching and B is related to bending. G^0 and B^0 stand for the state of the initial surface S_0.

Eq. 6 can be simplified and approximated [Terzo87a] [Grein96a], yielding to

$$E_{\text{int}}(s)=\int\sum_{i,j=1}^2 \alpha_{ij}\frac{d(s-s_0)}{di}\frac{d(s-s_0)}{dj}+\beta_{ij}\left(\frac{d(s-s_0)^2}{didj}\right)^2 ds \quad (7)$$

where α and β control the surface S shape. During a deformation between S_0 and S, α and β control surface's resistance to stretching and bending respectively.

Replacing the surface equation in Eq. 7 by our surface equation Eq. 5 yields to:

$$E_{\text{int}}(s) = \int \sum_{i,j=1}^{2} \alpha_{ij} (V^T - V_0^T) \frac{db}{di} . (V^T - V_0^T) \frac{db}{dj}$$
$$+ \beta_{ij} (\frac{(V^T - V_0^T) db)^2}{didj})^2 ds \quad (8)$$

Factorization of (V-V_0) can be done because of surface linearity and independence of control points relatively to parameters.

$$E_{\text{int}}(s) = (V - V_0)^T H (V - V_0) \quad (9)$$

H is a square matrix, built from the product of first order and second order derivatives of the B-Spline basis functions.

Notice that V_0 and V must be of equal dimension and must correspond to the same parametric subdivision.

3.2 Deformation external energy

Many external operators [Grein96a][Veltk95a] may be introduced to compose the total energy of the deformed surface. Here are briefly described some examples :

3.2.1 Point of attraction or repulsion

M is an attracting point of space, that attracts a piece of the surface S if the distance between M and every point of this piece is minimal. We get

$$E_{Att_{U,V}} = \int_U \int_V f(U,V) \left\| V^T b(U,V) - M \right\|^2 d_U d_V \quad (10)$$

where f(U,V)is a weight function.
Introducing the initial surface S_0 and developing, Eq. 10 becomes

$$E_{Att_{U,V}} = (V - V_0)^T \int_U \int_V f(U,V)\, b(U,V) \otimes b(U,V)\, d_U d_V\, (V - V_0)$$
$$-2(V - V_0) * \int_U \int_V f(U,V) * b(U,V) * (M - V_0^T b(U,V)) d_U d_V$$
$$+ \int_U \int_V (M - V_0^T b(U,V))^2\, d_U d_V \quad (11)$$

where $\otimes$ is the outer product.
We obtain

$$E_{Att_{U,V}} = (V - V_0)^T H (V - V_0) - 2(V - V_0) B + cst \quad (12)$$

In the same way, a repulsing point is expressed as the inverse of an attracting point.

3.2.2 Profiler

We introduce a new operator that force a piece of surface to mimic a B-Spline surface of reference (the profiler).

This tool has two main interests. In a modeling process, it allows to re-use predefined surfaces to build objects. In a reconstruction process, it allows to deform the surface according to data coming from objects surrounding the object to reconstruct.

We try to align the normals of the surface and of the profiler, which yields to :

$$E_P = \int_U \int_V \left\| n_S - n_P \right\|^2 d_U d_V \quad (13)$$

This constraint involves that the parametric domains (U,V) of the piece of surface to deform and of the profiler are identically subdivided.
An approximation of Eq. 13 consists in aligning tangent vectors at every point:

$$E_{P'} = \int_U \int_V \sum_{i=1}^{2} \left\| \frac{d(S)}{di} - \frac{d(M)}{di} \right\|^2 d_U d_V \quad (14)$$

U, V define the parametric space of the piece of surface and M the profiler.
Again introducing the initial surface and developing gives the i^{th} term :

$$E_{p'} = \int_U \int_V ((V - V_0)^T \frac{db}{du})^2 d_U d_V$$
$$-2 \int_U \int_V (V - V_0)^T \frac{db}{du} . (V_M - V_0)^T \frac{db}{du} d_U d_V$$
$$+ \int_U \int_V ((V_M - V_0)^T \frac{db}{du})^2 d_U d_V$$
$$= (V - V_0)^T \iint \frac{db}{du} \otimes \frac{db}{du} d_U d_V \quad (V - V_0)$$
$$-2 * (V - V_0)^T \iint \frac{db}{du} . (V_M - V_0)^T \frac{db}{du} d_U d_V$$
$$+ \iint ((V_M - V_0)^T \frac{db}{du})^2 d_U d_V \quad (15)$$

3.3 Constraints

3.3.1 "Point" constraint

So that the surface S contains a 3D point M_1 we search for a point $M \in S$ of parametric coordinates (u_1,v_1), (i.e M_1=V^Tb(u_1,v_1)) such that :

$$M_1 - V_0^T b(u_1,v_1) = V^T b(u_1,v_1) - V_0{}^T b(u_1,v_1) \quad (16)$$

This formulation corresponds to a linear constraint in the minimization. A possible choice for u_1,v_1 corresponds to the point of S_0 the closer to M_1 in order to make the deformation minimal.

3.3.2 "Curve" constraint

So that the surface S contains a 3D curve $C_E(t)$, we search for the curve c(u(t),v(t)) included in S :

$$\int_{C_0} (C_0 - C_E)^2 = \int_{C_0} (C_0 - C)^2 = 0 \quad (17)$$

where C_0 and C are respectively the compositions of S_0 and S with c, denoted S_0oc and Soc hereafter.

Each term of Eq. 17 may be replaced by the gradient expression :

$$\int_{C_0} (S_0 \circ c - C_E) \frac{\delta(S_0 \circ c)}{\delta V_0}\, dt = \int_{C_0} (S_0 \circ c - S \circ c) \frac{\delta(S_0 \circ c)}{\delta V_0}\, dt \quad (18)$$

Replacing the surface's expressions by the formulation of Eq. 5, we obtain:

$$V_0^T \int_{C_0} (b \circ c) \otimes (b \circ c)\, dt - \int_{C_0} C_E (b \circ c)\, dt = V_0^T \int_{C_0} (b \circ c) \otimes (b \circ c)\, dt - V^T \int_{C_0} (b \circ c) \otimes (b \circ c)\, dt \quad (19)$$

$$(V - V_0)^T \int_{C_0} (b \circ c) \otimes (b \circ c)\, dt = \int_{C_0} C_E (b \circ c)\, dt - V_0^T \int_{C_0} (b \circ c) \otimes (b \circ c)\, dt \quad (20)$$

Again we have a linear system of constraints on ($V-V_0$).

3.3.3 "Surface" constraint

Let $S_E = V_E^T b$ to be a 3D surface. For S_E to belong to S, we must verify :

$$\int_U \int_V (V - V_0)^T b\, d_U d_V = \int_U \int_V (V_E - V_0)^T b\, d_U d_V \quad (21)$$

To solve this constraint, we approximate it by discretizing on the parameter domain (U,V) to obtain a set of "point" constraints (see 3.3.1). The discretization level defines the precision with which this constraint is satisfied.

3.3.4 "Continuity" constraint

We have seen in a previous part that two rules must be respected to ensure the continuity between patterns.

If two adjacent patterns are of different levels of refinement, each boundary is constrained with a band of control points set to 0, which is equivalent to the following linear constraint :

$$(V_F - V_{0_F}) = -V_{0_F} \quad (22)$$

with V_F belonging to the boundary

If two adjacents patterns are of same level of refinement, the control points on the boundary of each pattern must be equal :

$$(V_{F1} - V_{0_{F1}}) - (V_{F2} - V_{0_{F2}}) = 0 \quad (23)$$

Notice that Eq. 22 assumes that continuity is ensured on the initial surface. If two boundaries of S_0 come from two separate refinements, the first rule must be applied.

4 CALCULATION AND HIERARCHY OF THE SURFACE

4.1 Calculation

We have seen in the previous part how to express the deformation energy. We have:

$$E_{Tot} = E_{\text{int}} + E_{Ext} \quad (24)$$

We must minimize
Min E_{tot} (25)
A ($V-V_0$)=b

For this minimization, we use linearly constrained quadratic optimization (from the IMSL library for instance).
We reduce the size of the system to minimize. For instance, surface refinement adds new "continuity" constraints. According to rule 1 of §2.2, control points set to 0 are removed from the system. According to rule 2 of §2.2, redundant equality constraints are removed (for instance $P_1 = P_2$ and $P_2 = P_3$ removes constraint $P_1 = P_3$). In order to implement this size reduction, we must know all the adjacent patterns of a given pattern. This is the reason why an arborescent representation of the surface hierarchy is used.

4.2 Surface hierarchy

To formulate the surface representation, we use a tree structure with a 4,n alphabet [Vajno97a] where n corresponds to the number of B-Spline patches belonging to a given pattern. The surface is then represented by a unique word. Several operations have been developed, such as tree development for the refinement process, and leaves display for the surface display. This structure allows the quick finding of a given pattern by a sequential course of the word representing the surface.

So we can easily know if a new constraint implies refining a pattern according to the number of constraints already influencing this pattern. This is particularly powerful when a pattern is submitted to many constraints, for instance for “curve” and “surface” constraints.

5 RESULTS AND CONCLUSION

Fig.3 to Fig. 7 show some examples of deformations with our model. Fig.3 show the initial surface S0, while Fig.4 show the surface deformed by a "point" constraint. In Fig.5 and Fig.6, we can see the result of an attraction point on the initial surface. In Fig. 7, we can see the result of a profiler on a piece of the initial surface. Here we can notice the locality of the deformation.

This method allows to quickly obtain various shapes of known objects, with an important flexibility for the user. Classical deformation operators available in the modeling literature can be easily added to the set of tools provided to the user. Moreover, the initial surface introduction spreads the use of our model to real world applications (3D reconstruction from 2D data, deformation control,...)

For now, the pattern used is a uniform B-Spline surface span, but the extension of this method to other non-uniform subdivisions is foreseen. Furthermore patterns fusion in order to reduce the number of unknowns has not been studied. The choice of parametric values influenced by a constraint or an operator must be automatized.

This method can also be used to compare different deformations. A deformation is represented by a tree structure. Building identical trees for two deformed surfaces and computing distances between each set of control points of a deformed surface and the set of control points of the initial surface helps to compare two different deformations.

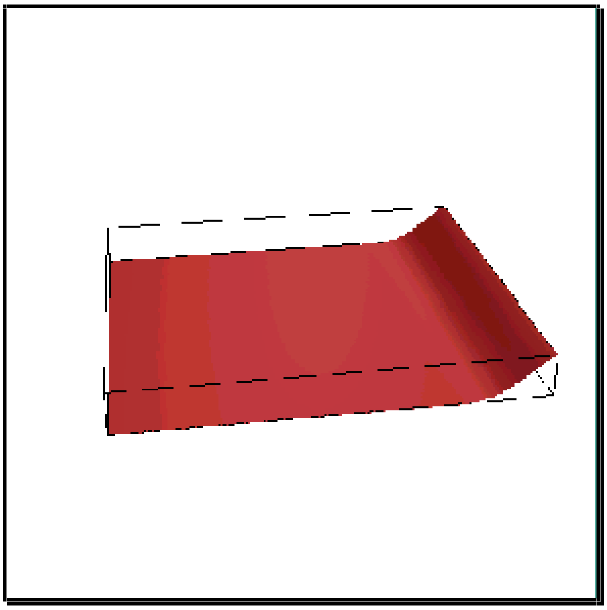

Fig. 3 Initial Surface

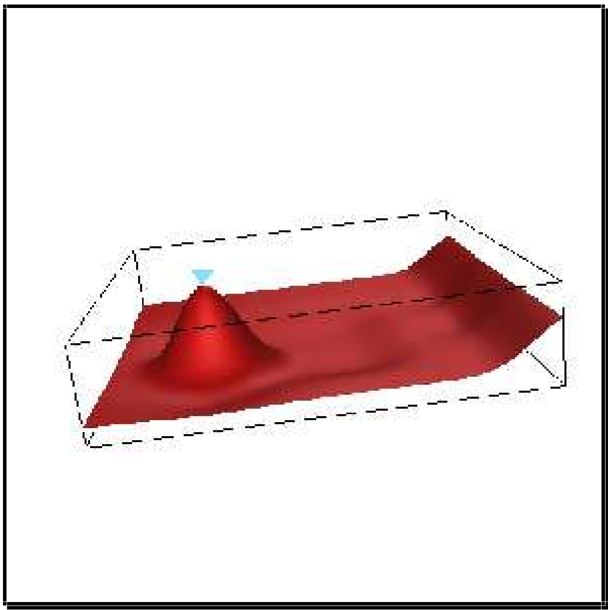

Fig. 4 Surface deformed by one "point" constraint

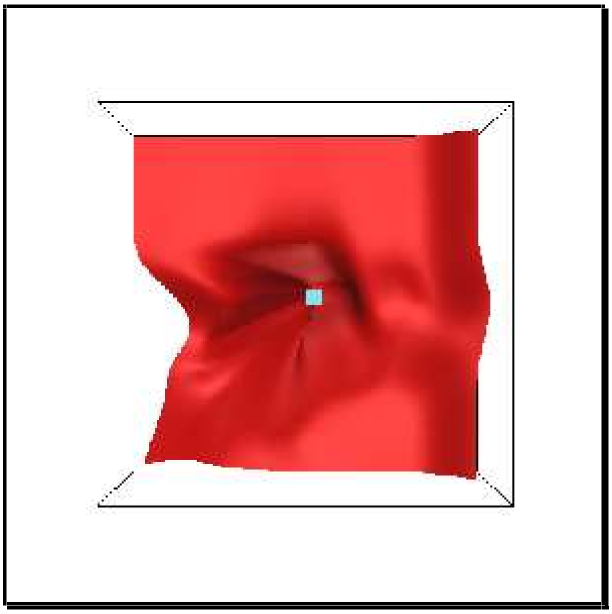

Fig. 5 Surface deformed by an attraction point

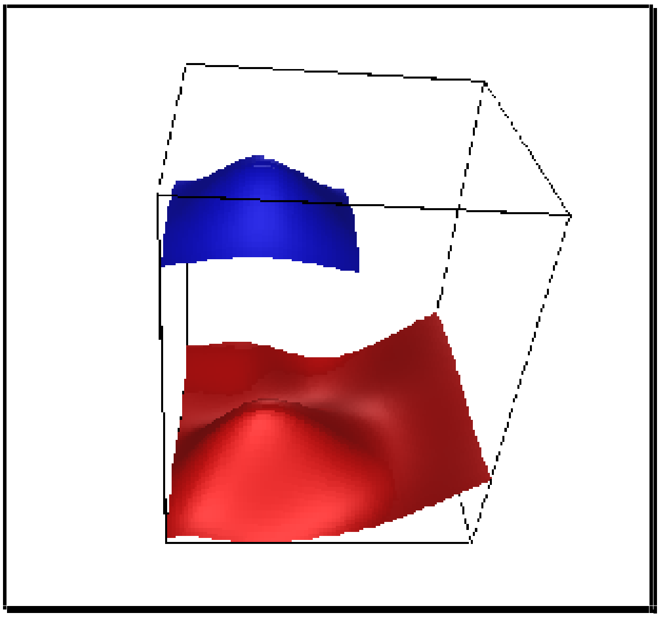

Fig. 7 Surface deformed by a profiler

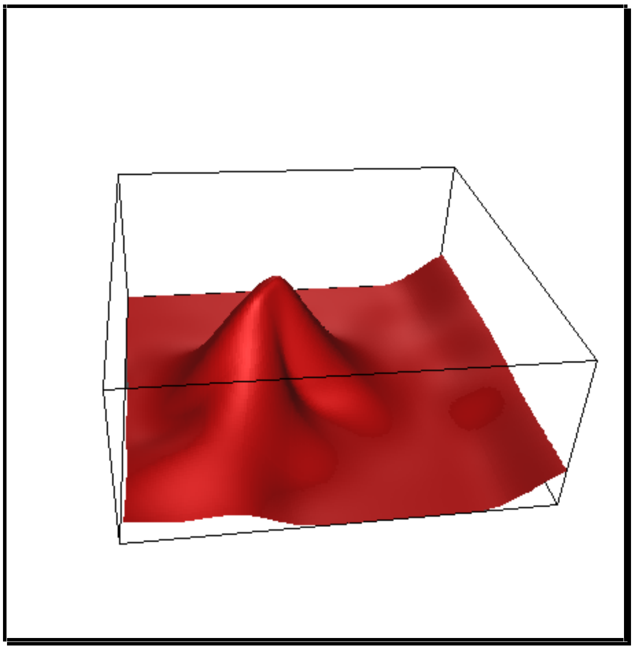

Fig. 6 Surface deformed by attraction point

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