PIXELS CLASSIFICATION IN NOISY DIGITAL PICTURES USING FUZZY ARITHMETIC

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ABSTRACT

This paper presents a new technique to extract, in noisy digital pictures, regions whose pixels fall, with a degree of uncertainty, in a given range of gray levels. The proposed method uses fuzzy numbers to describe in a compact way, at the early vision stage, the relevant information of the picture together with the uncertainty due to noise. This fuzzy model of the original picture is hence interrogated with a Marching-Cube-like algorithm to obtain, for a specified level of presumption, the pixels in a prescribed range. The quality of the obtained results is comparable with those obtained with more traditional, but less efficient, non-linear smoothing techniques.

1. INTRODUCTION

The construction and visualization of iso-curves is a standard technique for the investigation of bi-dimensional data. For example, the use of systems of iso-curves to gain insight into large collections of 2D geographic data predates the digital era. Similarly, in many cases, volumetric data can be efficiently and effectively presented through a sequence of iso-surfaces, or "shells", relative to successive density levels.

Construction and visualization of iso-surfaces, as regarded in this paper is an early vision task: further elaboration is needed to assign a semantic relevance to the detected surfaces. Iso-contouring, moreover, here takes into account only one of the features of each pixel at time (gray level, response to Sobel filter etc.). This grants efficiency but a price is paid to the accuracy of the classification. In applications that are not accuracy-critical iso-contouring is a viable alternative to more precise, but more costly, techniques based on neural nets or on extraction of principal features (eigenvalues, PCA etc).

In this paper we assume that for a digital picture or a digital volume an iso-curve, or an iso-surface, is the set of the pixels whose characteristics, like gray level, gradient magnitude etc. are constant, or in a prescribed range. Notice that, according to this definition, an iso-curve is not a one-dimensional set but a sub-region of the original picture (generally of elongated shape or "strip-like"): here the term "curve" is hence used somehow improperly. The classification of the pixels whose characteristics fall in a prescribed range or "window" is also known in literature as "windowing" and is widely applied. For example when, in the image under examination, objects and background are characterized by well separated values of gray, windowing is a simple, powerful way to detect the object/background boundary.

When the data (pixels or voxels) have been collected with a smaller sampling resolution than the resolution required in the visualization and the windowing range is reduced to a single value a popular approach to windowing is the Marching Cube Algorithm (MCA) [Loren87]. This technique assumes that the observed characteristic changes linearly from one point of the sample the closest sample points and makes use of few local rules to locate the segments (respectively polygons, in the 3D case) where the windowing value is attained.

Many variations of the Marching Cube Algorithm are known, but all seem to have two major source of difficulties: noise sensitivity and topological ambiguity. Even a small amount of spurious or erroneous data can confuse the windowing process destroying the topological integrity of the extracted region. Several heuristic rules have been proposed (see for example [Karro92]) to resolve topological ambiguities in applying the MCA, but none of them seem to avoid all the possible problems.

A common cure to noisy data is to apply a smoothing filtering like, for example, the non-linear median filter whose robustness to noise is rooted in the analogous property of the statistical median indicator. Median filtering, however, is computationally expensive and its use in real time processing of images, as required by many medical imaging procedures, is limited and becomes non practical for 3D data.

MCA, moreover, provides only a "hard" classification: points are classified as members of the approximating polyedral surface or outside of it and any information about the uncertainty coming from the noise in the original image is lost. The final user, hence, in many cases, has no clue about the validity of the results. It is clear that even a simple qualitative information of this kind can be extremely useful in the daily applications.

The simple algorithm described in this paper for bi-dimensional data tries to address the three above mentioned issues: noise reduction, topological soundness and uncertainty estimation, using Fuzzy Arithmetic.

According to the proposed approach, noise and uncertainty are naturally incorporated, in a controlled way, into a fuzzy model of the original image. The interrogation of this fuzzy model, done in a marching-cube-like fashion, provides in output regions where, with a given level of presumption, the pixels values fall in the required range. The regions relative to lower presumption levels naturally contain the regions relative to higher presumption levels: coloring these regions with a slowly changing LUT an immediate visual information, suggestive of the validity of the windowing process is provided.

The paper is organized as follows: Section 2, quickly reviews Fuzzy Arithmetic and reports the two fundamental steps of the proposed technique, i.e., the fuzzification procedure of the original image and its interrogation. Section 3 reports experimental results and compares them with similar results obtained with a non-fuzzy approach. The paper concludes with a summary and with some notes about further related researches.

2. FUZZY WINDOWING IN DIGITAL IMAGES

2.1. Fuzzy Arithmetic

In order to make this paper self-contained in this subsection we review some fundamental concepts of Fuzzy Arithmetic. Definitions and results not explicitly mentioned here can be found in [Zimme91], [Anile95]. A fuzzy real number F is an interval $[a,b]$ of the real line together with a "membership function", $m(t)$ from the set of the real numbers to the unit interval $[0,1]$ such that:

i) $m(t) = 0$ for t in $R \setminus]a,b[$;
ii) There is at least a point c in $[a,b]$ such that $m(c) = 1$.

$m(t)$ can be a general function, however, for the application of this paper only triangular fuzzy numbers are used. A triangular fuzzy numbers $F = ([a,b],m(t))$ is a fuzzy number such that there is only one point c in $[a,b]$ such that $m(c) = 1$ and the function $m(t)$ is linear and monotonically increasing from a to c and linear and monotonically decreasing from c to b. Examples of two triangular fuzzy numbers are diagrammed in Fig 1.(a).

Given a real number s in $[0,1]$ the interval F_s is the subinterval $[l_s,h_s]$ of $[a,b]$ such that for every t in $[l_s,h_s]$, $m(t)$ is greater or equal to s. F_s is said "s-cut" of the number F.

A fuzzy number F can be equivalently assigned as a pair $([a,b],m(t))$ or as a family of intervals F_s with s in $[0,1]$ such that if $s>t$, F_s is contained in F_t.

Making use of s-cuts it is possible to introduce arithmetic operations over fuzzy numbers: if OP is a binary operation over real numbers the interval $[a,b]$ OP $[c,d]$ is the interval defined as $[min(t\ OP\ s), max(t\ OP\ s)]$ with t in $[a,b]$ and s in $[c,d]$.

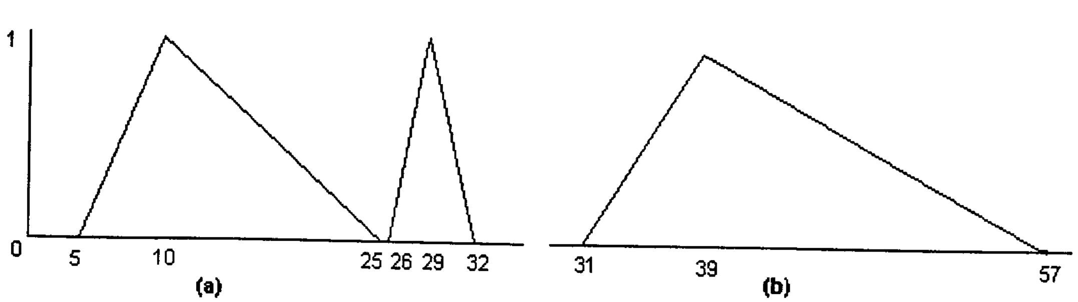

Fig.1 (a) Triangular fuzzy numbers with bases [5, 25] and [26, 32] and vertexes 10 and 29; **(b)** the sum of the two fuzzy numbers of Figure 1.(a).

If *OP* is a binary operation over the reals *F OP G* is the fuzzy numbers described by the collection of *s*-cuts F_s *OP* G_s with *s* in *[0,1]*.

If *OP* is a linear operation triangular fuzzy numbers generate a new triangular fuzzy number. This is not necessarily true if *OP* is non-linear. Fig.1.(b) shows the sum between the two fuzzy numbers in Fig.1.(a)

Linear interpolation between two fuzzy numbers *F* and *G*, *L(F, G, t)*, is defined over the unit real interval and takes value over the set of the fuzzy numbers. More precisely, in terms of *s*-cuts:

$$L_s(F, G, t) = F_s (1 - t) + G_s t$$

In Fig.2 some *s*-cuts of the fuzzy numbers interpolating the triangular numbers with bases *[0,6]* and *[6,9]* and vertexes *3* and *7.5*, are diagrammed.

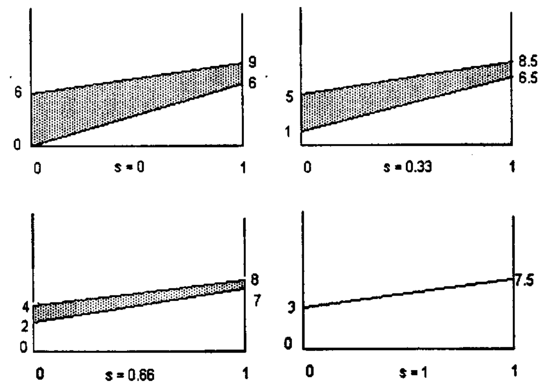

Fig. 2 The linear fuzzy interpolation of the two fuzzy triangular numbers with bases [0, 6] and [6, 9] and vertexes 3 and 7.5.

A possible extension of the windowing procedure to the fuzzy case can be defined as follows. Given a presumption level *s* and a range *[a,b]*, the set:

$H_{[a,b],s}$ = *{t* in *[0,1]* such that $L_s(F_s, G_s, t)$ and *[a,b]* have a non-empty intersection*}*

is the set where the fuzzy function *L(F, G, t)* assumes, with presumption *s*, values in *[a,b]*.

The fuzzy subset $H_{[a,b]}$ of the unit interval, defined with the *s*-cuts $H_{[a,b],s}$ is the answer to the windowing query of *L(F, G, t)* with respect to the range *[a,b]*.

The results of querying the fuzzy function in Fig.2 with the range *[2,4]* are shown in Fig.3 for the presumption levels *s=0*, *s=0.33*, *s=0.66* and *s=1*.

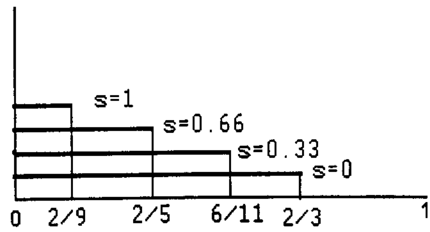

Fig.3. *s*-cuts of the answer to the query of the function in Figure 2 with respect to the query [2, 4].

2.2. Fuzzy digital pictures

The first step of the proposed procedure is to obtain a fuzzy version of the original data set.

Let ***h*** be a positive integer. Let ***M*** x ***N*** be the dimension in pixels of the original picture. The fuzzy version of the picture is a lattice of ***M/h*** x ***N/h*** data point. . In the experiments the parameter ***h*** has been equal to *30* or *20*.

To each point *(i,j)* of the lattice, *i = 1..M/h*, *j = 1..N/h*, can be assigned a triangular fuzzy number as follows:

1. consider the square sub-region of the original picture of side ***2h*** pixels, whose center is the pixel *(ih,jh);*
2. compute the median, *MED*, the first quartile, *FQ*, and the third quartile, *TQ*, of the values of the $4h^2$ pixels in the square;
3. assign to the point *(i,j)* the fuzzy triangular number whose basis is the *interval [FQ, TQ]* and with vertex *MED*.

This simple procedure provides a, somehow qualitative, statistical summary of the original picture and uses indicators that are well known for their robustness to noise and outliers. The fuzzy data in the lattice realize, indeed, a form of data reduction through a sub-sampling of the original data set, maintaining, at the same time, minimal information about the spatial variability.

Notice that the status-of-the-art methods in the analysis of spatial data [Cress96],[Viert96], [Kruse87], provides generally more information than the simple approach taken here. However this is achieved with a greater computational effort. We found experimentally that the proposed fuzzy technique balance well the complexity issues with the precision required for the present application.

2.3. Interrogation of the fuzzy data set

In this subsection we show how to interrogate the fuzzy summary obtained in the previous subsection.

The kind of query considered here is a pair *([a, b],s)*. The first component of the query is a real interval: we are looking for regions of the image whose pixels, prior the degradation due to noise and errors, assume presumably values in the range *[a, b]*; the second component of the query, *s*, is a real number in *[0, 1]* and represents the presumption level required.

The query is processed with an approach that is reminiscent of the MCA. This is a reasonable choice because the fuzzy image has a far lesser resolution that the original picture and, in most situations, local linearity can be safely assumed.
The interrogation algorithm considers, one at the time, the unit rectangles of the lattice of fuzzy data.

Let *UL* (Upper Left) be the fuzzy number associated with the vertex *(i,j)* of the rectangle.
Let *UR* (Upper Right) be the fuzzy number associated with the vertex *(i + 1, j)* of the rectangle.
Let *LL* (Lower Left) be the fuzzy number associated with the vertex *(i, j + 1)* of the rectangle.
Let *LR* (Lower Right) be the fuzzy number associated with the vertex *(i+1, j + 1)* of the rectangle. For an example see Fig.4.

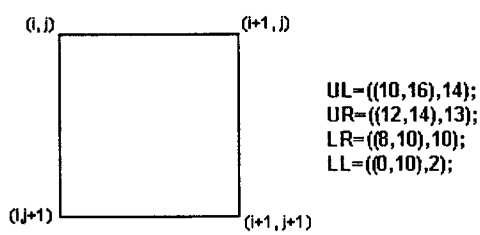

Fig.4 An example of a rectangle of the lattice. The fuzzy triangular numbers are assigned as pairs (base, vertex).

The linear interpolations *L(UL, UR, t)*, *L(UR, LR, t)*, *L(LR, LL, t)* and *L(LL, UL, t)* are queried, at the presumption level s, with the interval *[a, b]* as shown in subsection 2.1. As a result one obtains at most four sub-segments of the edges joining the points *(i, j)*, *(i + 1, j)*, *(i + 1, j + 1)*, *(i, j + 1)* of the lattice. The answer to the original query, inside the rectangle, is obtained as the convex hull of the sub-segments. The union of these convex hulls over all the rectangles in the lattice provides the output of the query. The process is illustrated in Fig.5 for few values of *s*.

3. EXPERIMENTAL RESULTS

The procedure in Section 2 has been applied to three kind of pictures: echocardiograms, text with uniform noise and gradient magnitude of some test image. For comparison the picture have been, in alternative, smoothed with a median filter and windowed with a traditional "hard" classification. Some results are shown in Fig 6 to Fig.9.

A relevant difference is in the efficiency of the process: traditional median filter applied to all the pixels in the image is much more time consuming than the computation of the fuzzy summary of the picture. Notice that this summary, in the proposed method, requires median computation as well, but only for a very small subset of the pixels in the original image.
If a box of 5 times 5 pixels is used for median smoothing the proposed procedure requires about half of the time required for traditional median filtering, while, if the smoothing box is as large as 15 time 15 pixels, the proposed procedure requires about ten time less computing time than the traditional median filter. For example, in our, implementation, traditional processing with median filter of Fig.7 required about 69 seconds, while fuzzy processing required 7 seconds.

Moreover while to extract semantically satisfying sub-regions the proposed method needs very narrow ranges (generally a single threshold value is sufficient), traditional smoothing and windowing requires significatively larger, harder to optimize, ranges.

The fuzzy approach, finally, provides very good visual clues to the quality of the segmentation obtained. Coloring with an appropriate LUT the regions extracted at different presumption levels a user is presented in a single picture the results of several interrogation and discrimination and analysis is hence greatly simplified as Fig.9 dramatically shows.

4. Conclusions

We have described a new approach, based on fuzzy arithmetic, to pixel classification in digital pictures. The proposed method provides results of comparable or better quality than similar more traditional windowing techniques. Its low computational cost makes the technique a good real

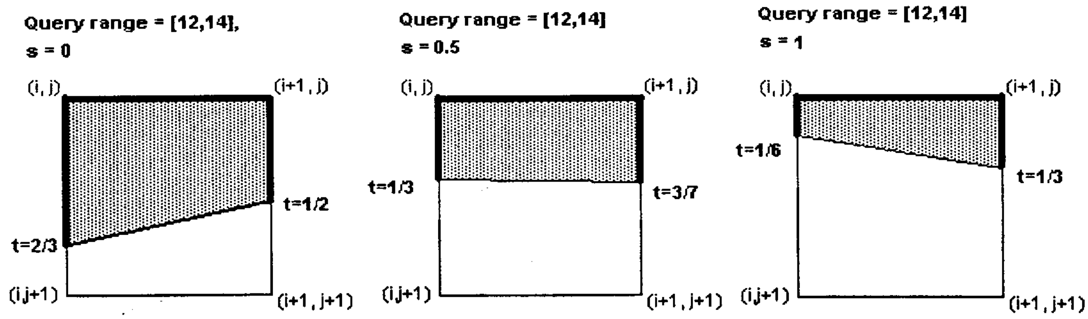

Fig.5 Results of some interrogations of the rectangle in Fig.4

time tools for image analysis and enhancement. A great advantage of the algorithm is in the ability to provide qualitative information about the validity of the results obtained incorporating in a simple and natural way statistical information about data variability and uncertainty. As shown in the example this can be an invaluable tool for practical purposes in real time medical image processing.
The new algorithm can be extended in straightforward way to higher dimensions and research in this direction is in progress.

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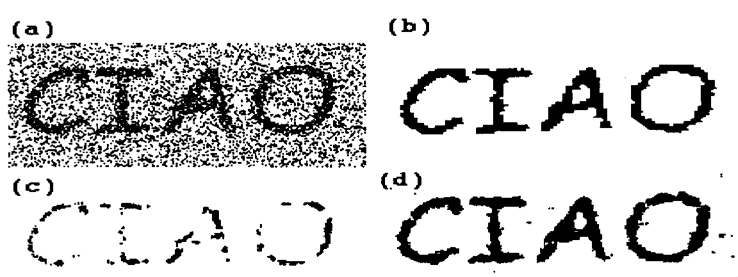

Fig.6 **(a)** A fragment of text with added uniform noise; **(b)** the output of fuzzy classification for s = 0.8 relative to the range [0,0]; **(c)** the output of hard classification of the median filtered image relative to the range [0,0]; **(d)** the output of hard classification of the median filtered image relative to the range [0,100].

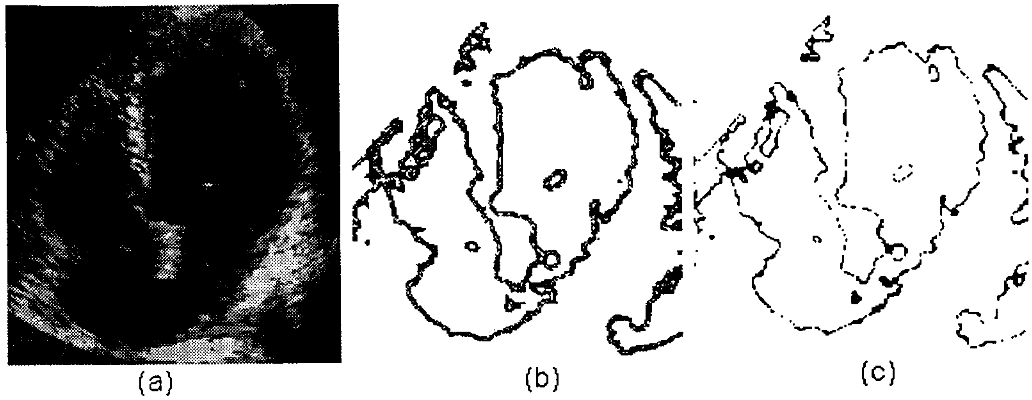

Fig.7. **(a)** Original echocardiography; **(b)** fuzzy classification relative to the window [38,38]; **(c)** hard classification of the median filtered image relative to the window [36,40].

Fig 8. The gradient magnitude picture of the standard "Lena" photograph has been computed. **(a)** The gradient has been fuzzy windowed relative to [128,128]; **(b)** the same gradient windowed relative to [128,128].

Fig.9 An enlarged copy of the same echocardiography of Fig.7 with the fuzzy classified pixel reproduced with a slow changing LUT overlaid to the original picture.