3D underground reconstruction for real-time and collaborative virtual reality environment

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Abstract

Urban planning has become a relevant issue to achieve the sustainable development of the modern cities. It requires ubiquitous systems including 3D GIS capabilities and high performance without many hardware requirements. This paper deals with the development of a web application to visualize, interact and explore, through 3D perspective, the underground infrastructures of our college campus. Its 3D reconstruction raises many challenges which remains to be surmounted. The main aim of this research is the 3D modeling of underground infrastructure in order to real-time analytics of its current features. In addition, this system supports virtual reality technology to interact with the GIS data into a collaborative environment available to be performed with mobile platforms.

Keywords

Virtual Reality, Underground features, 3D GIS, Real-time visualization and interaction

1 INTRODUCTION

Urban underground space has been playing a very important role for urban development in recent years. The subsoil contains many types of features which offer a wide number of services in the city. They are usually placed at different depths so it is likely to find overlapping infrastructure elements. This is the main reason why underground must be explored through a 3D perspective in order to acquire an accurate inspection from different viewpoints of the scene. In addition, the underground infrastructures are not directly visible so their maintenance tasks become imprecise and inefficient. This trouble means challenges for 3D underground reconstruction and the extraction of its features because of its digitalization is usually outdated and incomplete.

Nowadays, three-dimensional Geographic Information Systems (3D GIS) are typically used for 3D visualization and management of virtual cities. They are applied in many professional domains like urban planning, environmental impact and resources efficiency. However, GIS users are not usually satisfied with the performance of these platforms because of the high hardware requirements for 3D rendering. In order to provide an efficient solution in this context, we have developed a custom application for 3D underground representation which includes some 3D GIS capabilities (Figure 1). Thus, this system provides complex analysis and 3D description of spatial objects and underground features. It is a web-based system which allows network access from different remote areas. Users can synchronize the current underground data and update the information system through a wireless network or a 3G access. For this reason, we have defined a client-server architecture in which the spatial database is allocated in the server and the rendering of scene is launched in the client device. The development environment in 3D is based on WebGL and the code has been optimized to be adapted to multiple mobile devices. Moreover, the usage of the WebVR API provides us support for exposing virtual reality devices in web applications. This feature has been added for underground exploring through HTC VIVE VR headset [1].

The type of source data and the fact that these features are not visible involve the challenging task of the 3D reconstruction. Currently, these infrastructures are normally depicted in 2D by means of CAD layers without any topological relationships. 2D mapping implies a high number of overlapping layers which require a upper abstraction level to identify each object. This trouble has been faced in this research work and we have applied preprocessing techniques to achieve a 3D representation. The main features of our solutions are: (1) 3D underground reconstruction, (2) virtual reality environment, (3) real-time navigation and interaction, (4) ubiquitous system adapted to different mobile devices, (5) spatial database with topological relationships. In addition, during the development of this application we have made a study with the finality of finding the best technique for modeling the terrain surface. The placement of these infrastructure is intrinsically linked to the terrain relief. Our approach is based on a precise digital elevation model (DEM) from LiDAR data used to

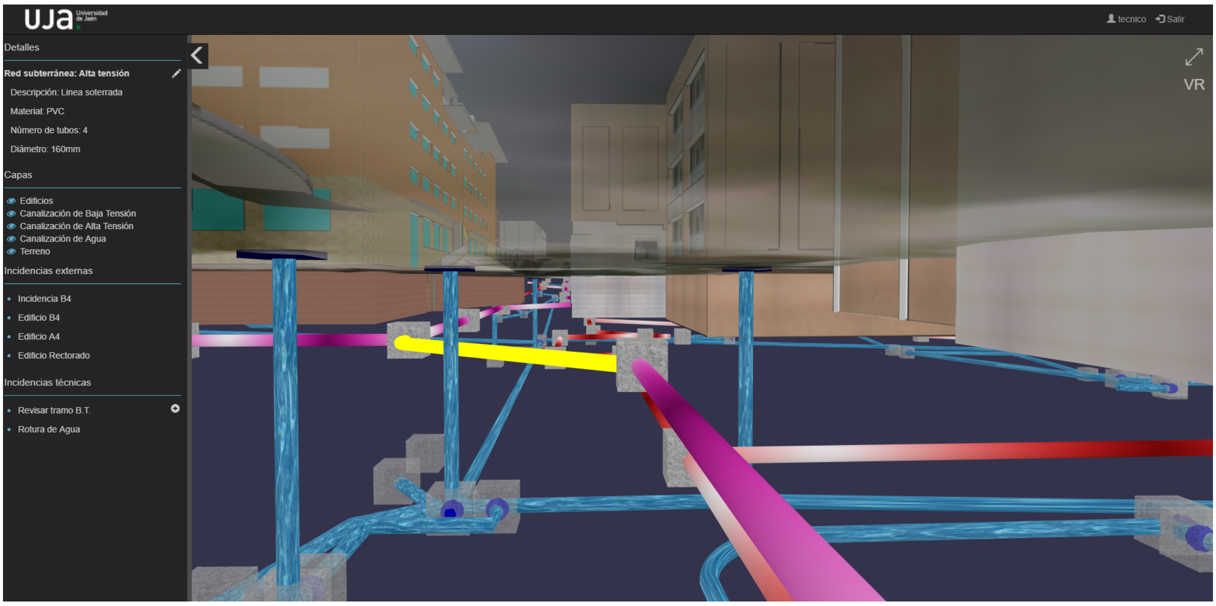

Figure 1: The 3D GIS user interface. Underground infrastructure is represented in the virtual environment. Haptic devices are used to interact with a tubing (yellow) to review his specs. Panel control is designed (left) to review the information, layers occlusion and maintenance reports.

calculate the location of the underground features according to CAD information [2].

In this paper, we first summarize the state of the art regarding GIS tools, web-based solutions for 3D mapping and acquisition methods of urban surfaces. Then, we focus on the study of the underground infrastructure before describing the steps followed for 3D reconstruction in our web-based framework. Afterwards, the results obtained and the assessment of novel contributions are discussed.

The main contributions of this paper are:

- An efficient method for 3D reconstruction of underground features and data model design.
- A web-based solution for real-time underground exploring through virtual reality into mobile platform.

2 RELATED WORK

According to 3D GIS tools there are many research projects which describe their utilities for remote sensing, forest health studies or hydraulic simulations. Recently, open source GIS [3] are known to provide user-friendly interfaces to manage and visualize 3D city models. Most of these frameworks offer a reliable representation when adding depth values to this geo-data as well as enhance their visual features. 3D GIS capabilities are focused to process large 3D models or point clouds. For this purpose, there are several frameworks and stand-alone software packages that provide some semantic and volumetric 3D models such as GRASS [4] or a popular data model like CityGML [5]. Grass includes the NVIZ visualization suite which is capable of rendering 2D/3D raster and vector maps. In this context, CityGML is a popular open standardized data model used to represent 3D cities and landscapes. The aim of CityGML is to reach a common definition and understanding of the basic entities, attributes and relations within a city model. Although both frameworks have added 3D functionality to manage city models, there are not web-based tools to efficient rendering of underground features into a collaborative workflow. In regards to proprietary GIS platforms and 3D modeling software we highlight ArcGIS [6], City Engine [7]. However, currently they are still missing some features for accurate 3D modeling, visualization and management of underground networks. However, all of these solutions are so heavy systems in order to achieve a high performance in mobile devices and the underground modeling is not supported.

Web GIS has played a relevant role in supporting collaborative environment for analysis and visualization of geospatial data on the Internet. It is considered a viable solution for gathering and sharing of collected data from various case studies [8]. The advances of web-based geo-information systems provide remote access from any location, 3D mapping and spatial analysis in real-time. In this context, the GPU acceleration is used to achieve an adequate performance for a large 3D models [9]. ArcGIS allows us to build full-featured 3D applications powered by web scenes that can include different information layers such as terrain, integrated mesh scene layers and 3D objects. Cesium [10] is a popular open-source Javascript library focused on creating the leading web-based globe and map for visualizing dynamic data. This framework provides a complete Earth imagery support and the capability to visualize 3D models in virtual environments. Another interesting web framework is iTowns [11], which provides a visualization of 3D geographic data and precise measurements. This project supports different types of data allowing the visualization of street view images and terrestrial LiDAR point clouds. Nevertheless, all of them require so high hardware requirements to render the large size of 3D models and point clouds. Moreover, their low performances in mobile devices and the bad usability of user interfaces are enough reasons to reject the use of them for our solution.

The quality of 3D GIS tool depends on the proper choice of a precise acquisition method for surface reconstructions. Traditional techniques for creating Digital Elevation Model (DEM) are very costly regarding time consuming because of the land surveying. Today, LiDAR data and photogrammetry techniques has become one of the major methods to generate 3D ground models. Recently, LIDAR sensors on board the UAVs have become a powerful way to produce a DEM files due to the very effective data acquisition by the small distance sampling. In this paper, three acquisition methods have been explored to get a precise digital elevation model: terrestrial laser scanning (TLS), photogrammetry technique using aerial imagery that is captured with UAV (Unmanned Aerial Vehicle) and LiDAR-PNOA data provided by the Institute Geographic National [12]. Firstly, TLS stations has great potential in creating a high resolution and dense point clouds of the ground surface. However, in urban spaces there are a huge amount of buildings with their roofs and vegetation that may occlude important geometric features which require of manual setting and tedious capture processes. Secondly, UAVs have become common practice in getting visual information like images used in different fields application, such as 3D mapping, structure monitoring and cultural heritage documentation. These systems are equipped with optic (RGB) cameras oriented toward different angles to generate accurate elevation models. Finally, airborne Light Detection and Ranging (LiDAR) systems are also popular techniques in remote sensing area for accurate 3D reconstructions.

In this paper, we present a web application coded in Javascript/WebGL for the visualization and interaction of 3D underground infrastructure in real-time and distributed networking environment. It is based on Babylon.js engine and thus, supporting post-processing, controls, 3D models, animations and more features.

3 UNDERGROUND MODELING

The main aim of our study is to define the methodology for the 3D underground reconstruction through a web-based information system. It provides an accurate spatial features inventory and 3D tools for collaborative analysis and real-time interaction.

3.1 Topological spatial database design

The 3D inventory of underground utility has been created following a topological data model. The spatial database stores vectorial entities as well as their topological relationships. However, CAD files contain many objects which must be pre-processed because of their non-connected geometry. We have designed a PostGIS [13] database located in the server. It adds support for geographic objects allowing location through SQL querying. PostGIS is an extension of PostgreSQL and is released under the GNU General Public License, offering many spatial functions rarely found in other competing databases such as Oracle and SQL Server.

The underground infrastructures, that are studied in this research work, can be divided into two main layers: sewer/water and electrical wiring. In order to classify CAD files, our spatial database is composed of thematic tables that represent each underground layer. The structure defined in each table is formed by the follow attributes: ID (primary key), material, type of geometry (vector entities), diameter of pipes, number of tubes and the absolute location of geometry. Thus, topological relationships among underground entities allows us to identify influence areas for specific underground failures.

3.2 Processing of CAD layers

CAD files are the most common representation for designing urban infrastructures. Nowadays, there are not remote sensing technologies which offer an efficient techniques for geo-location of underground utility networks. A significant investment in the subsoil detection, positioning, and documentation management is currently in progress.

In this paper, we have studied the vectorial CAD layers and entities of the underground representation. Frequently, this information is stored in 2D CAD files. This data cannot be directly used for our 3D application due to the lack of geo-referenced maps and because of the incoherencies found in the input data. The result of processing this metadata is stored in a PostGIS database in order to support efficient spatial querying. For this purpose, firstly we have studied the input data to find an effective method to classify and process the whole CAD information.

In order to process the CAD file is necessary to make the follow steps: (1) the geolocation and classification of the underground layers, (2) the repair of each unconnected pipe (3) and finally the simplification of redundant geometry. Consequently, we manage vectorial data such as points, lines, polylines and polygons.

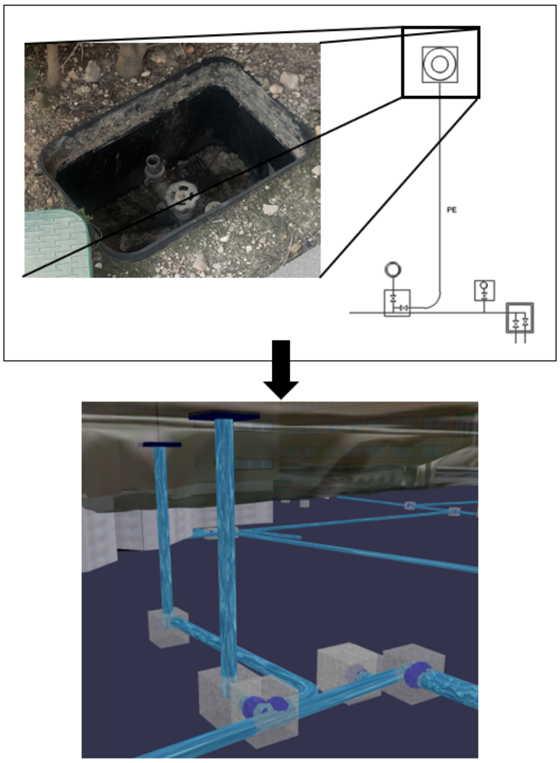

Figure 2: The enhanced visualization of our 3D GIS approach

These primitives symbolize specific entities of the real world like buildings, sewers, pipes, control stations or irrigation taps. We have used MapInfo Pro [14], which provides automatic utilities to edit the geometry and transform the vector data before being uploaded into the database server.

The input data, at first, is geo-located using three ground control points (GCPs). It makes possible the underground infrastructure mapping over the digital terrain model. Afterwards, CAD layers are classified based on the underground structure which they represent. In this work we have managed four types of entity groups: buildings, sewer network, overhead power line and low voltage wires. Secondly, the incoherent CAD objects must be repaired in order to create and manage right topological entities. Lines and polylines that represent underground pipes are usually unconnected because of drawing CAD errors. These cases are detected and solved through a method coded in MapInfo Pro. This algorithm creates new polylines if the end point of one line is close to the start point of the following. Finally, we simplify CAD layers by removing all polygons, which represent buildings, control station or some irrigation taps. They are replaced by their centroids where the corresponding 3D models are going to be subsequently located. As a result, we have made a total conversion of the original CAD map to achieve the 3D formal data structure in order to be represented in our virtual environments. Figure 2 shows the appearance improvement achieved in order to visualize the underground infrastructure reconstruction.

3.3 Digital surface modeling

Underground modeling keeps a close relationship with the relief terrain. In order to determine an accurate measure of the depth of the infrastructures, it is necessary to know the surface unevenness. In fact, our 3D GIS framework includes a Digital Elevation Model (DEM) whose vertical accuracy must be lower than 50 centimeters. A research project is carried out for generating a proper DEM of our college campus, and thus the 3D mapping of its underground infrastructure (Figure 3). For this purpose, we have used raw LiDAR data and aerial imagery from UAV.

Recently, photogrammetry has been widely used to create 3D maps and 3D models from images. We have placed a camera on the drone pointed vertically towards the ground. Multiple overlapping photos (80% overlap) of the ground are taken with UAVs through a programmed flight path. However, the terrain model is a complex and huge mesh which requires many computational efforts to be rendered in a web-based environment. In addition, there are many places of the ground which are occluded by trees and buildings. These problems have been solved using LiDAR-PNOA data. This data source is provided by PNOA project [15] which is leaded by the National Geographic Institute of Spain. This point cloud contains (X,Y,Z) coordinates, with 0.5 points/m^2, being the vertical precision lower than 20cm RMSE Z. The LiDAR information provides the capacity of calculating the Digital Elevation Model (DEM). As a result, in our application the terrain model is generated through the height map from the LiDAR point cloud. It has been calculated using Global Mapper tool in order to acquire the elevation grid surface and is exported as raster image.

3.4 Web-based 3D engine

WebGL enables a direct integration of 3D graphics into standard web pages [16]. Thereby, web applications are capable of integrating hardware-accelerated 3D graphics in network environments. It provides a co-management of heterogeneous data between client devices in order to discover, create and share 3D information. There are popular web virtual globes such as Google Maps, Apple Flyover, or OpenStreetMap [17] focused on rendering massive real-world terrain. They are composed by Digital Terrain Model (DTM), imagery and vector datasets and some 3D city landscape. The massive rendering streaming of huge 3D city models implies latency problems which must be considered in any web-based development [18]. Nevertheless,

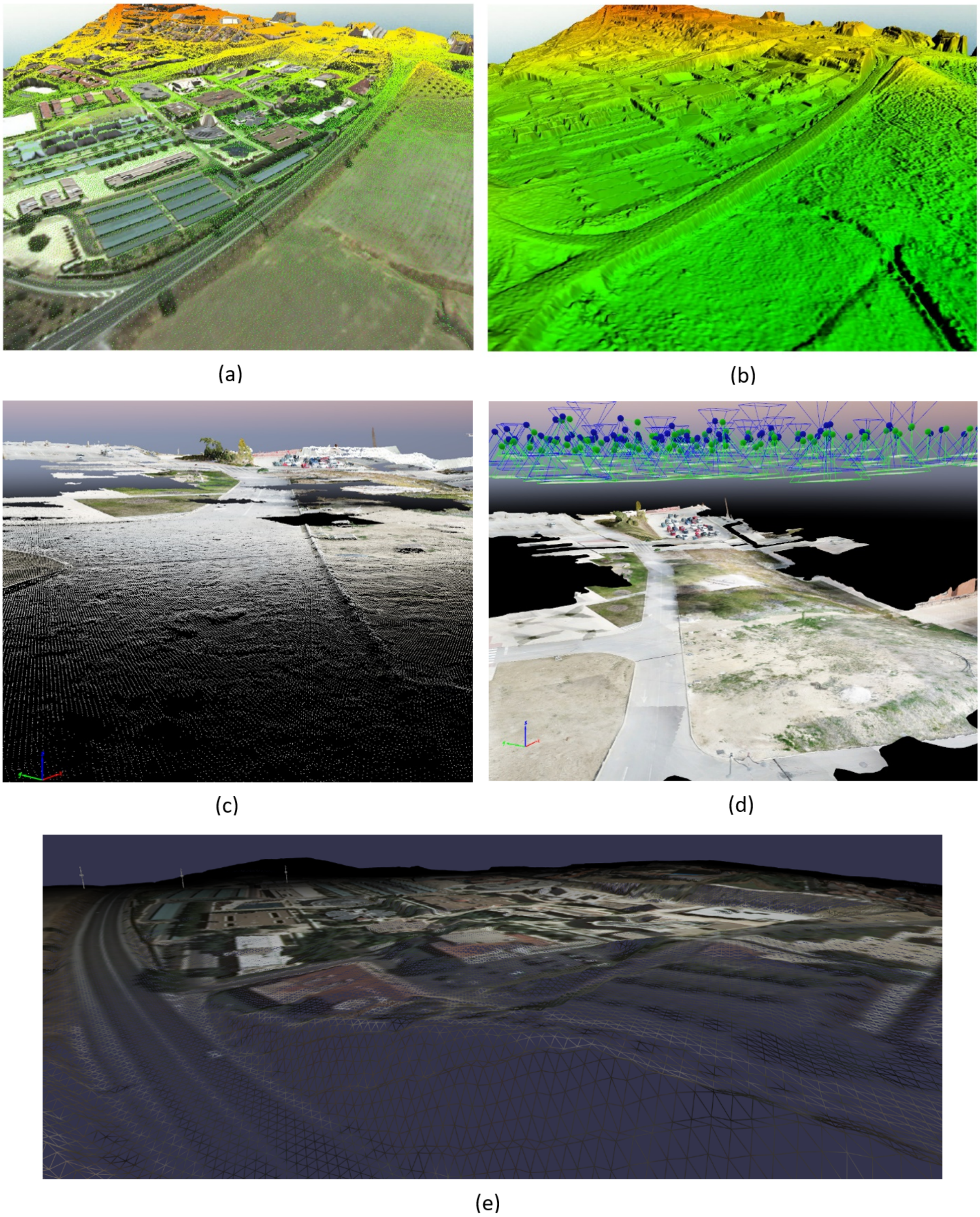

Figure 3: The relief surface modeling. LiDAR point cloud is mapped to the terrain and is processed for DEM calculation (a-b). UAV flight over the college campus and the generation of a huge mesh (c-d). As a result, the ground is calculated through LiDAR elevation model and is depicted in our application (e).

there is not any extended 3D web framework to facilitate the management of the underground structure.

The high resolution of the ground elevation model to allocate the 3D position of underground features needs efficient code for an adequate performance. The complex ground mesh rendered into our web application must be simplified in order to reduce the number of triangles generated. The 3D models in our virtual environment must be explored without lag effect, over 60 frames per second. These requirements are satisfied to guarantee the best performance into mobile platforms. Our system is based on BabylonJS engine [19]. It is an open source 3D engine coded with WebGL and Javascript to render interactive 3D computer graphics an 2D graphics within any compatible web browser. The last version of BabylonJS contains important improvements like WebGL v.2 and WebVR v.1.1 support, a better performance for invisible Solid Particle System (SPS) and PBR rendering techniques aim to simulate real life lighting. During the development of our 3D graphic scene, we have carried out the following features: the height maps to generate realistic grounds, the use of Solid Particle System (SPS), the octrees to optimize the collisions calculation and the WebVR camera to provide virtual reality navigation around the environment reconstruction.

The rendering of a huge terrain mesh requires a high computational effort to perform the scene. Instead of importing a 3D ground model, the terrain is generated by a height map. It is a grayscale image whose pixel's color is interpreted as the distance of the displacement or height from the floor. As a result, the ground is a triangulated mesh which is defined with a specific number of subdivisions. It increases the complexity of the mesh in order to improve the visual quality. In our case, depending on the target device, the ground subdivisions are changed to ensure the best performance.

Another main utility of our framework is linked to the SPS feature. It is an updatable particle system composed by separate and different geometry forms. Each particle has the same properties than any other mesh. The use of this particle system has made possible an efficient rendering of the underground infrastructures. If each pipe was represented as a unique object, the total number of meshes was very high and the frame rate dropped quite suddenly. A specific underground layer has one particle system which contains sewerage networks, electrical control stations and other 3D entities. As a result, our scene simultaneously manages until four particle systems.

The last enhance is focused on the virtual reality requirements. One of the costliest feature is the collisions detected with the 3D models of the scene. It has been optimized using a tree data structure that can improve the selection of entities based on space coordinates. In our scene buildings cannot be crossed during the navigation process, then collisions must be computed. The use of octrees optimizes the selection of sub-meshes belonging to the buildings for detecting quickly collisions. Afterwards regarding virtual reality navigation, the camera setting must be modified. Babylon 3.0 supports WebVR API 1.1 specifications in the latest version of Microsoft Edge, Chromium and Firefox. We add the functionality of VRcamera in our 3D environment to explore underground utility, through virtual reality, from any location with mobile platforms.

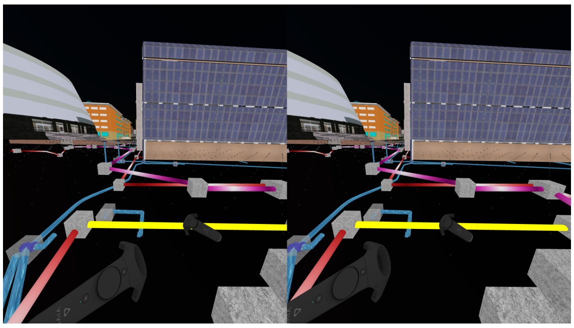

Figure 4: Virtual reality environment through HTC VIVE glasses

3.5 Bringing Virtual Reality to the Web

Real-time interaction with 3D models and 3D navigation around the scene for exploring the underground features are overcome challenges. Moreover, the efficient scene rendering allows the visualization of the 3D virtual environment without any decrease of the performance in mobile devices. Current web GIS tools provide 3D representation of spatial data, but without VR features. Virtual reality is playing an important role in many computer vision applications for a direct interaction with GIS data [20]. This technology provides immersive environments and therefore its usage for underground exploring provides a realistic interaction with the infrastructures (Figure 4). Our VR approach involves an innovative solution for visualization and analysis in the field of urban planning. It definitely helps to acquire a 3D perception of these infrastructures and plan the future urban growing, taking advantage of the complete knowledge of underground utilities.

Our framework is a web-based solution focused on real-time analytics of the 3D underground features for mobile platforms. Thus, it is necessary to apply optimization techniques to improve the performance of the rendering process, specially in virtual reality environments. In addition, some latency issues usually present in these type of web applications must be resolved. This is the reason why the meshes of the scene have been reconstructed, using simplification methods, in order to reduce the size of 3D models. In the testing process, we have used HTC Vive headset to interact and visualize the 3D environment due to the perfect behavior of its haptic controllers. The VR display quality is one of the most important components of virtual reality headsets. However, the frame rate needed for rendering virtual reality applications is an important restriction. It must be high enough to prevent motion sickness and provide a smooth experience. In this context, we have applied the following optimization techniques to assure 60 frames per second (fps).

- **Virtual camera setup:** Firstly, in order to improve the performance of the 3D environment we have reduced the view field of camera. The (X, Y, Z) planes have been delimited until 30 units.
- **View Frustum culling:** In order to render only the 3D models visible by the camera we have applied frustum culling technique. View Frustum Cullers (VFCs) are typically used in virtual reality applications [21]. This method provides a significant improvement of the performance because only the 3D models inside this volume are rendered in the scene.
- **Octrees implementation:** The collision calculation requires an important computational effort [22]. In this case, the use of octrees in the 3D buildings meshes reduces the time required for the collision detection.

4 DISCUSSION OF THE RESULTS

This paper presents an innovative web-based application (Figure 5) for real-time interaction and visualization of 3D underground infrastructure of urban spaces through virtual reality. This application involves a continuous refinement model that combines an integral spatial database to store the geo-location of the subsoil objects and the descriptive information of the underground infrastructure. This system has a set of tools for 3D inspections, navigation, interaction and analysis on-site where there is any underground fault. In general, these infrastructures cannot be directly visualized. The contribution of virtual reality provides the possibility of a direct interaction with the underground features. This technology allow us to acquire a realistic perception of underground and a high knowledge of its current features. For this, we have chosen HTC Vive headset due to the accurate interaction of its haptic controllers. As a result, the immersive experiences, during the underground exploration, raises a novel way to analysis these infrastructures. In addition, we have described different acquisition methods for ground model generation. LiDAR-PNOA has been the data resource chosen for our application. The LiDAR point cloud has a high horizontal and vertical accuracy and provides a precise height map which is required to create the ground model in our application. It makes possible an accurate

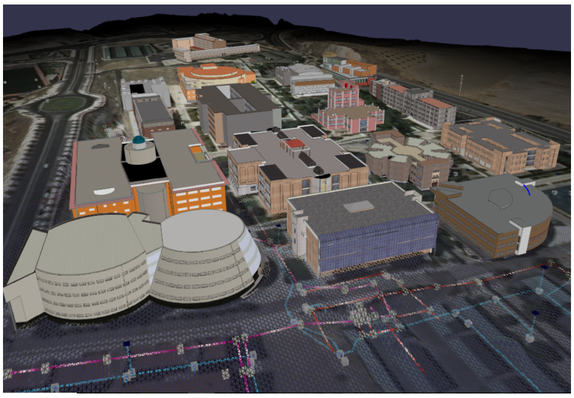

Figure 5: 3D GIS environment

3D mapping of underground infrastructure with the terrain relief and the calculation of its depth. In this paper, we have studied the input CAD information, the maintenance reports and we have generated an accurate ground elevation model our college campus. Based on this data, we have developed a 3D virtual environment to represent and manage the underground infrastructure providing as well a collaborative workflow. Thus, the users can also realize of the changes carried out by others at the same moment, due to the real-time update of the database information available in the application.

5 CONCLUSIONS AND FUTURE WORK

Underground infrastructure is the focus for many urban planning studies. Its facilities form the backbone of the progress and welfare of the modern cities. In the last few years, there has been a growing recognition about the benefits due to the accurate geolocation of underground infrastructure. The deterioration of these utilities needs a routine maintenance which must be proactive rather than reactive emergency response. The study and analysis of the growing underground facilities are challenges of many research works due to the complex data structure and the inability of their direct inspections. The resource optimization and the correct planning of resources consumption in the cities are some of the current issues that must be analyzed. In this paper, we have described the features of our web-based system in order to share the effective methodology followed for 3D underground management. The main goal is its 3D reconstruction in order to real-time interaction and study its current features.

In this paper, we propose an innovative 3D environment which provides a real-time interaction, visualization and management of underground infrastructure into a collaborative web-based application. During many years, we have studied the underground features and the best way to monitor them. Today, we show a system based on WebGL which has been optimized to be performed from mobile platforms. In addition, it includes a spatial database in order to store the whole descriptive information of the underground features. Our three-dimensional web-based system opens new lines for collaborative communication between planners and provides a reliable interaction with the underground features through virtual reality. Currently, we are working in a continuous increment of 3D underground representation to support 4D analysis and thus predicting the future needs for the smart city maintenance.

6 ACKNOWLEDGMENTS

This work has been partially supported by the Ministerio de Economía y Competitividad and the European Union (via ERDF funds) through the research projects TIN2014-58218-R and TIN2017-84968-R.

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