Optimal Fuzzy Aggregation of Secondary Attributes in Recognition Problems

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ABSTRACT
A method of optimal fuzzy aggregation of secondary attributes in problems of recognizing a person from images is proposed. The primary attribute is a characteristic feature of an object, allowing to distinguish one object from another. The secondary attribute is a result of recognition algorithm, based on primary attributes. Fuzzy integrals of Sugeno and Choquet are used as aggregation operators of secondary attributes. Optimality of aggregation here stands for the selection of the best solution of recognition problem by means of fuzzy integrals. It also indicates conditions, when the use of Sugeno fuzzy integral as an aggregation operator is better than the use of Choquet fuzzy integral in recognition problems. The experiment results are presented.

Keywords
Person recognition, computer vision, information fusion, fuzzy integral.

1. INTRODUCTION
The junction, aggregation or fusion of information – is one of the most substantial aspects for construction of state-of-the-art intelligence system [Aka05, Kwa05, Pop06].

Pattern recognition systems, scene analysis systems, image processing systems, and computer vision systems use aggregation of data from various information sources to make proper decision. Information source is meant to be a certain object recognition algorithm. The data, produced by this algorithm, allows to recognize the object, i.e. refer it to some given object class. The tendency to merge several information sources in order to get final solution, results from the fact that each source, taken separately, might have high ambiguity, or imprecision of data, which reduces with the aggregation of sources [Wan04].

Since the practical use of probabilistic approach to aggregation does not always give good results [Val00, Rao05], the methods of data aggregation, based on the theory of fuzzy integrals (especially integrals of Choquet and Sugeno) are getting more popular. Generally, fuzzy integral is a nonlinear functional, defined with a certain fuzzy measure [Sug77]. Application of fuzzy integral, unlike the probabilistic methods, takes proper data, received from various information sources, and importance of these sources and its subsets, in the very process of aggregation.

The recognition problem statement for a sphere of computer vision given the presence of several information sources is the following: If $T = \{t_1,...,t_m\}$ - is a set of classes, describing the object A, found at image, then the recognition problem is to determine class t_k, $1 \le k \le m$ of object A, and to evaluate the extent of confidence, that object A refers to class t_k, given some object data is received by means of different object recognition algorithms.

The paper [Liu01] deals with traffic dynamic analysis, using the fuzzy integral as an operator of aggregating the information, extracted from image at various levels of abstraction. High values have been received not only for recognition of car on the road, but also for prediction of traffic trajectories and accident. The paper [Aka05] presents the automated path selection systems for pedestrians. By means of fuzzy integral, this system takes into account individual preferences of system's clients for the travel line, and the objective evidence of the road itself. The high-precision recognition of vehicles at battlefield with aggregation of information by means of fuzzy integral is presented in [Tah90].

Aggregation of information to recognize the person by the image of its face was described in [Kwa05], and [Che05]. The aggregated information in the first work was received by wavelet-fragmentation of image. The aggregated information in the second work was derived from the main information source (video camera), and from the additional source (infrared sensor) as well.

Choquet fuzzy integral is preferred to Sugeno in the majority of these articles, but this paper will show the conditions, where Sugeno integral is optimal. Furthermore, the algorithm of recognizing person by its image, based on Sugeno fuzzy integral, used as an operator of aggregating information from several data sources, will be reviewed herewith. The primary attribute is characteristic feature of object, allowing to distinguish one object from another. The secondary attribute is a result of recognition algorithm, based on primary attributes. Therefore, the problem of optimal fuzzy aggregation is to select the best solution of recognition, calculated by means of Sugeno and Choquet fuzzy integrals.

In section 2 definitions of fuzzy integral and fuzzy measure are described. Section 3 deals with recognition problem solution algorithm using Sugeno fuzzy integral as an aggregation operator. The statement evidence of optimal fuzzy integral selection is stated in Section 4. Section 5 is devoted to the experimental verification of statement adduced.

2. FUZZY MEASURES AND FUZZY INTEGRALS

Definition 1: A measure m is a set function $m: P(X) \rightarrow R$ with properties:

1) $A \subseteq X \Leftrightarrow m(A) \geq 0;$

2) $m(\varnothing) = 0;$

3) If $A, B \in P(X)$, then $m(A \cup B) = m(A) + m(B) - m(A \cap B).$

, where X - arbitrary set, $P(X)$ - set of all subsets of X, R- set of real numbers.

Definition 2: If X is arbitrary set, β is δ-algebra of subsets of X. Then set function $g(\cdot)$, in the form $g: \beta \rightarrow [0,1]$, is called fuzzy measure with properties [Sug77]:

1. Limitation: $g(\varnothing) = 0, g(X) = 1;$

2. Monotonicity : if $A, B \in \beta$ and $A \subset B$, then $g(A) \leq g(B)$;

3. Continuity: if $\{F_n\}$ is monotone sequence and $F_n \in \beta$, then $\lim\limits_{n \to \infty} g(F_n) = g(\lim\limits_{n \to \infty} F_n)$.

(X, β, g) is measurable space with fuzzy measure. Fuzzy measure is not necessarily additive: $g(A \cup B) \neq g(A) + g(B)$.

Because the fuzzy measure of the union of two disjoint subsets cannot be directly computed from the component measures, the g_λ-fuzzy measures were introduced, satisfying the following additional property [Tah90]:

4. $\forall A, B \subset X$ and $A \cap B = \varnothing$, $g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B)$, for some $-1 < \lambda < +\infty$.

Using the notion of fuzzy measures, the concept of the fuzzy integral was defined.

For measurable space (X, β, g) and continuous function h with properties:

1. If $x \leq y$, then $h(x) \leq h(y); x, y \in R$

2. $\lim\limits_{x \to -\infty} h(x) = 0;\ \lim\limits_{x \to +\infty} h(x) = 1.$

Definition 3: Sugeno fuzzy integral [Tah90] of the function $h: X \rightarrow [0,1]$ over set $A \subseteq X$ with respect to a fuzzy measure $g(\cdot)$ is defined by:

$$S_g(h) = \int_A h(x) \circ g(\cdot) = \max_{i=1}^{n} [\min(h(x_i), g(A_i))]$$

, where $h(x_1) \geq h(x_2) \geq ... \geq h(x_n)$. (1)

Definition 4: Choquet fuzzy integral [Kwa05] of the $h: X \rightarrow [0,1]$ over set $A \subseteq X$ with respect to a fuzzy measure $g(\cdot)$ is defined by:

$$C_g(h) = \int_A h(x) \circ g(\cdot) = \sum_{i=1}^{n} (h(x_i) - h(x_{i+1}) g(A_i))$$

,where $h(x_{n+1}) = 0$ and $h(x_1) \geq h(x_2) \geq ... \geq h(x_n)$.(2)

Definition 5: Let $X = \{x_1, ..., x_n\}$ be a finite set, and x_i is information source, and then degree of importance of the information source $x_i \in X$ is $g^i = g(\{x_i\})$.

If fuzzy measure g is g_λ-fuzzy measure, then the values of $g(A_i)$ can be found of recursion:

$g(A_1) = g(\{x_1\}) = g^1$, (3)

$g(A_i) = g^i + g(A_{i-1}) + \lambda g^i g(A_{i-1})$, for $1 < i \le n$.

According to lemma 1 [Tah90], for a fixed set of $\{g^i\}, 0 < g^i < 1$ there a unique $\lambda \in (-1, +\infty)$ exists and $\lambda \ne 0: \lambda + 1 = \prod_{i=1}^{n} (1 + \lambda g^i)$. (4)

Therefore, λ can be found from equation (4), and in fact, one needs only to solve an *(n - 1)*st degree polynomial and find the unique root greater than - 1.

3. SOLUTION OF RECOGNITION PROBLEM

Solution of recognition problem by means of recognition algorithm, based on Sugeno or Choquet fuzzy integral, is the following:

If $X = \{x_1, x_2, ..., x_n\}$ - set of object recognition algorithms, A - object being recognized, $h_k : X \to [0,1]$ - secondary attribute, denoting the result of calculating membership of object A in class t_k, $h^{\mu}{}_k(x_i)$ - secondary attribute after the fuzzification, i.e. the transformation to fuzzy form by means of certain membership function $\mu(h\ {}_k(x_i))$. Accordingly, $h^{\mu}{}_k(x_i)$ is an indicator of our confidence, that object A belongs to class t_k, using algorithm x_i. Here 1 stands for absolute confidence, that object A belongs to class t_k, and 0 stands for absolute confidence, that object A does not belong to class t_k.

In addition, each x_i is corresponded with extent of significance g^i, which denotes the degree of importance of information source x_i for recognition of class t_k. These values may be set up by expert or calculated from training data.

Then, object recognition algorithm with the use of data aggregation will be given by Figure 1. The database record is created for each object, which may be recognized by the system. Records are processed in series, and a fuzzy integral is calculated for each record. After all, the record with maximum value of fuzzy integral is selected. This record serves as class t_k, the recognized object belongs to. It is necessary to note, that Choquet fuzzy integral also can be used in this algorithm.

```
BEGIN
  Calculate degree g^i for each recognition
  algorithm x_i ∈ X = {x_1, x_2 ..., x_n}
  FOR each record in the database K=1,…,N;
      N-number of records
    Calculate value h^μ_k(x_i)
    Calculate λ_k ∈ (−1,+∞) using equation (4)
    IF NOT h^μ_k(x_1) ≥ h^μ_k(x_2) ≥ ... ≥ h^μ_k(x_n)
    THEN Reordering X = {x_1, x_2 ..., x_n} in
           accordance with the condition
    ELSE  Calculate g_k(A_i) using the recursion (3)
           Calculate fuzzy integral (1)
  END FOR
  Find the record with maximum S_k(h)
END
```

Figure 1. Recognition algorithm based on Sugeno fuzzy integral.

4. OPTIMAL SELECTION OF FUZZY INTEGRAL

Statement. Provided that the information sources x_i on object A give high-precision recognition result, i.e. the majority of sources give maximum, or minimum result of recognizing $h(x_i)$; the use of Sugeno fuzzy integral as an aggregation operator is preferred to Choquet, to aggregate information from this sources by means of algorithm 1.

Evidence.

Primary argumentation relies on investigations, conducted in [Bol96,Gra06a], proving that Choquet fuzzy integral presents generalization of arithmetical mean notion, while Sugeno integral presents generalization of median (if three or more information sources are used). Therefore, given the equal weights proportion (values of fuzzy measure) in this evidence, the values of fuzzy integrals will be nearly equal to values of arithmetical mean and median for Choquet and Sugeno fuzzy integrals accordingly.

Regarding optimization problem as a problem of finding the best possible solution, we may determine that:

If U – set of solutions of recognition problem, $u_j \in U$ - solution calculated with fuzzy integral (*j=1* - Choquet integral, *j=2* - Sugeno integral), the efficiency function $G(u_j)$ shall be maximized.

Hence, with *n=l+m,* where *n* – quantity of information sources x_i, i.e. the quantity of algorithms used in object recognition;

l - quantity of sources giving maximum result $h_{\max}(x)$,

m - quantity of sources giving minimum result $h_{\min}(x)$,

where $h_{\max}(x) > h_{\min}(x)$. We must evidence that $G(u_2) > G(u_1)$ will be true given *l>>m*, and $G(u_2) > G(u_1)$ given *l<<m*.

(For simplicity $h^{\mu}(x_i)$ will be noted as $h(x_i)$)

Case 1. *l>>m*

Choquet fuzzy integral is given with:

$$C\ (h) = \frac{h_{\max}(x_1) + h_{\max}(x_2) + \ldots + h_{\max}(x_l) + + h_{\min}(x_1) + h_{\min}(x_2) + \ldots + h_{\min}(x_m)}{n} =$$

$$= \frac{\sum_{t=1}^{l} h_{\max}(x_t) + \sum_{v=1}^{m} h_{\min}(x_v)}{n};$$

Median is a value of sorted series, dividing the series into two equal parts, so that 50% of “lower” series items shall have the values not exceeding median, while “upper” 50% items – values equal to median or exceeding it. Therefore, Sugeno fuzzy integral is given with:

$$S\ (h) = \{h_{\min}(x_1), h_{\min}(x_2), \ldots, h_{\min}(x_m), h_{\max}(x_1), h_{\max}(x_2), \ldots, h_{\max}(x_l)\}$$

So, since *l>>m*, median value is $S\ (h) = h_{\max}(x_p)$, where *p=1,..,l.*

It is obvious that having $h_{\max}(x) > h_{\min}(x)$ we receive

$$h_{\max}(x_p) > \frac{\sum_{t=1}^{l} h_{\max}(x_t) + \sum_{v=1}^{m} h_{\min}(x_v)}{n};$$

Hence, $S\ (h) > C(h)$, i.e. provided that the majority of recognition algorithms refer the object to given class, the aggregated value of Sugeno fuzzy integral will exceed the value of Choquet fuzzy integral, that is why the usage of Sugeno integral is optimal, i.e. $G(u_2) > G(u_1)$ Q.E.D.

Case 2. *l<<m*

Choquet fuzzy integral is given with:

$$C(h) = \frac{\sum_{t=1}^{l} h_{\max}(x_t) + \sum_{v=1}^{m} h_{\min}(x_v)}{n};$$

and Sugeno fuzzy integral here is $S\ (h) = h_{\min}(x_p)$, where *p=1,..,m.*

Having $h_{\max}(x) > h_{\min}(x)$, we receive

$$h_{\min}(x_p) < \frac{\sum_{t=1}^{l} h_{\max}(x_t) + \sum_{v=1}^{m} h_{\min}(x_v)}{n};$$

Thus, $S\ (h) < C(h)$. I.e., in semantical meaning, when the majority of recognition algorithms do not refer object to the given class, the aggregation value of Sugeno fuzzy integral will be less than of Choquet fuzzy integral. Therefore, the usage of Sugeno integral in this case is optimal, i.e. $G(u_2) > G(u_1)$ Q.E.D.

It must be noted, that this statement is right, when there are several reliable or high-precision recognition algorithms, which are aggregated with one or two not so effective algorithms. The possible situation might be that these algorithms cannot be excluded from the aggregation process due to external conditions: high-precision recognition result, archived with this algorithm, based on determining some primary attributes of object, which appear occasionally;

or due to internal conditions: algorithm is in positive interaction with one or several algorithms from reliable group. It means that to find the final evaluation of object, the importance of algorithms pair is greater than the importance of each algorithm taken separately [Gra00b].

5. APPLICATION

To verify the statement adduced, some experiments on recognition of person by its image have been conducted. To recognize a person, these experiments use three recognition algorithms: Hidden Markov Model (HMM) – modification of OpenCV implementation [Ope01], Color Definition Algorithm (CDA) – own implementation, Relationship Estimation Algorithm (REA) – own implementation of [Kwo99].

Each algorithm must be trained for all users that have to be recognized. For that, the database was created, consisting of records of users, “familiar” to the system. Figure 2 depicts the database, holding necessary information for recognition algorithms.

Figure 2. The database of users.

Hidden Markov Model is built (trained) for stored image of every user's face in gray scale. Then a new image is sent to its input. The result of modeling process is a probability, denoting the extent of conformity between trained and input images. The difference of probabilities HMM $h(x_1)$ is a difference of probability, calculated for stored image (P_{old}), and probability, calculated for input recognized image (P_{new}):

$$h(x_1) = | P_{old} - P_{new} |$$

If the difference exceeds the threshold (determined experimentally, and equal to 2), then the user does not belong to given record in the database.

Color Definition Algorithm determines the value of color (by three color channels r-red, g-green, b-blue) of user's "shirt"; compares this value with the color value of user being recognized (or it can be texture of "shirt"). If Euclidean distance $h(x_2)$ exceeds the threshold (determined experimentally, and equal to 40), the user is considered not to belong to this record. Euclidean distance $h(x_2)$ is given by:

$$h(x_2) = \sqrt{(r_{old} - r_{new})^2 + (g_{old} - g_{new})^2 + (b_{old} - b_{new})^2}$$

Relationship Estimation Algorithm determines distance between eyes (S^1), eyes and nose (S^2), nose and mouth (S^3), eyes and mouth (S^4), eyes and chin (S^5); and calculates Euclidean distance $h(x_3)$ to user being recognized, according to ratios of these distances. If the threshold (determined experimentally, and equal to 10), is exceeded, the user is considered not to belong to this record. Euclidean distance $h(x_3)$ is given by:

$$h(x_3) = \sqrt{(S^1_{old} - S^1_{new})^2 + (S^2_{old} - S^2_{new})^2 + (S^3_{old} - S^3_{new})^2 + (S^4_{old} - S^4_{new})^2 + (S^5_{old} - S^5_{new})^2}$$

In order to calculate values $h^{\mu}(x_i)$, *i=1,2,3*, values $h(x_i)$, calculated with algorithms, have been fuzzificated by means of half-trapezoid membership function for HMM:

$$h^{\mu}(x_1) = \begin{cases} 0, h(x_1) \geq 2 \\ \dfrac{2 - h(x_1)}{2 - 1.5}, 1.5 < h(x_1) < 2 \\ 1, 0 \leq h(x_1) \leq 1.5 \end{cases}$$

triangular function for CDA:

$$h^{\mu}(x_2) = \begin{cases} 0, h(x_2) \geq 40 \\ \dfrac{40 - h(x_2)}{40}, 0 < h(x_2) < 40 \\ 1, h(x_2) = 0 \end{cases}$$

and triangular function for REA:

$$h^{\mu}(x_3) = \begin{cases} 0, h(x_3) \geq 10 \\ \dfrac{10 - h(x_3)}{10}, 0 < h(x_3) < 10 \\ 1, h(x_3) = 0 \end{cases}$$

Graphs of membership functions are presented on Figure 3.

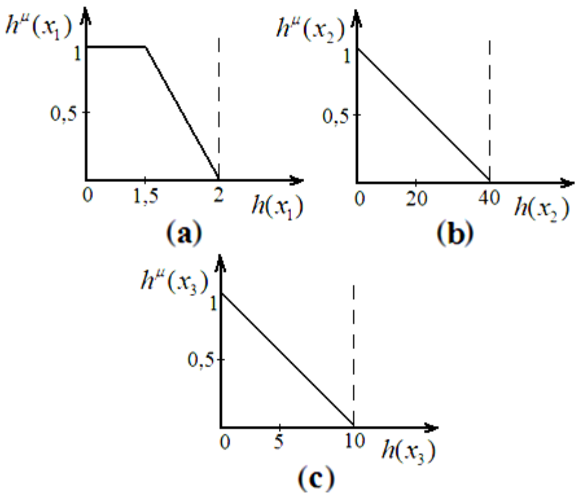

Figure 3. Fuzzification of algorithms values (a)-HMM, (b)-CDA, (c)-REA.

Table 1 contains the most typical results (from 50 experiments) of calculating Choquet and Sugeno fuzzy integrals by means of algorithm 1. The second, third and fourth columns represent values $h^{\mu}(x\ )$ of HMM, CDA and REA algorithms. The fifth and sixth columns contain calculated values of Choquet and Sugeno fuzzy integrals. Last column contains the name of integral, which reaches optimal aggregation value under such input values $h^{\mu}(x\ )$.

Most algorithms in first experiment produce high value of new user's belonging to the given database record. Sugeno integral has the maximum result, that is why this integral allows to get more optimal solution of recognition problem in this case. The same is fair for experiments three and four.

Most algorithms in the second experiment show that the user does not belong to the given record that is why the result of aggregation should be minimum. In this case Sugeno integral also produces more optimal recognition result, thereby confirming the statement, prescribed in section 4.

Results of experiment five are not so unambiguous. In this case Choquet fuzzy integral may be optimal, as well as Sugeno fuzzy integral. This depends on the extents of reliability g^i of used recognition algorithms and values of fuzzy measures, calculated with formulas (3).

Therefore, the method of optimal fuzzy aggregation of secondary attributes for object recognition problems in image, consists of the following steps:

I. Selection of object recognition algorithms (selection of secondary attributes).

II. Selection of fuzzy integral, having most optimal value for given secondary attributes.

III. Decision of recognition problem using chosen secondary attributes and fuzzy integral by means of algorithm 1.

6. CONCLUSION

In paper the method of optimal fuzzy aggregation of data from different information sources to recognize object by using fuzzy integral is presented. Fuzzy integral provides effective and natural fusion of data from different information sources.

It also indicates conditions, when the use of Sugeno fuzzy integral as an aggregation operator is more optimal than the use of Choquet fuzzy integral. Conducted experiments proved the statement adduced by the real example of recognizing person by his image. In future work more experiments will be described to show the performance of the proposed method.

Now this method is used in intuitive interaction

№	$h^{\mu}(x_i)$ values			Sugeno fuzzy integral values ($S(h)$)	Choquet fuzzy integral values ($C(h)$)	Optimal fuzzy integral
	HMM	CDA	REA			
1	0,9	0,9	0,1	0,86	0,9	$S(h)$
2	0,1	0,1	0,9	0,18	0,1	$S(h)$
3	0,9	0,9	0,8	0,89	0,9	$S(h)$
4	0,8	0,8	0,7	0,79	0,8	$S(h)$
5	0,5	0,4	0,3	0,48	0,4	$C(h)$ or $S(h)$

Table 1. Results of aggregation by means of Choquet and Sugeno fuzzy integrals.

system "Person – TV set" for robust recognition of TV users. Person recognition with high accuracy and stability (because of several recognition algorithms processing together) provides granting and restriction opportunities of users. The system can automatically grant opportunities for the known user (list of best channels, special color, brightness and volume settings) and can restrict it (access only to a few channels for some users).

The development of method with the use of additional information sources is planned later on. Such systems may additionally use algorithms, based on audio or infrared sensors.

Figure 4. Intuitive interaction system "Person – TV set".

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