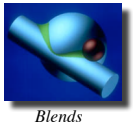
Georgia Tech College of Computing Interactive & Intelligent Computing

MAGIC lab Modeling, Animation, Graphic, Interaction & Compression

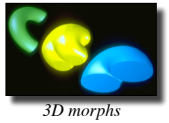
GVU center georgia tech

Shape comparison, simplification, & compression: **Simplifying smooth shapes**

Jarek Rossignac
Georgia Tech



Blends



3D morphs

Complexity measures for 2D shapes

Many measures of complexity are useful in different contexts:

- **Transmission**: compressed file size
  - **Kolmogorov**: Length of data and program
- **Processing**: Number of bounding elements
- **Algebraic**: Polynomial degree of bounding curves
- **Visibility**: Number of guards needed
- **Graphics**: Number of intersections with "random" ray

We focus primarily on morphological ones:

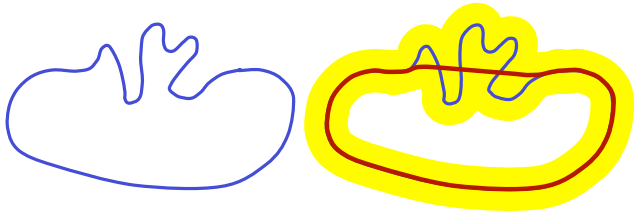
- **Sharpness**: curvature, high frequency energy
- **Regularity**: Minimal feature size
- **Topology**: Number of components, holes (, genus in 3D)
- **Tightness**: Perimeter length / surface area

Morphological Simplification

Jason Williams & Jarek Rossignac
GVU Center, IRIS Cluster, and College of Computing
Georgia Institute of Technology
www.cc.gatech.edu/~jarek

The road-tightening problem

- By law, the state must own all land located at a distance less then or equal to r from a state road.
- The states owns an old road C and wants to make it r-smoother.
  - Wand the radius of curvature to be > r everywhere
- Can it do so without purchasing any new land?
  - For simplicity, assume first that C is a manifold closed loop.



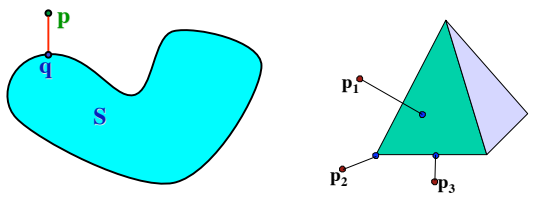
Distance and closest projection

$D_{Min}(\mathbf{p},\mathbf{S})$ is the **minimum distance** from point **p** to set **S**.

The **closest projection** $C_S(\mathbf{p}) = \{ \mathbf{q} : \|\mathbf{pq}\| = D_{Min}(\mathbf{p},\mathbf{S}) \}$

If the boundary of **S** is smooth, vector **qp** is orthogonal to **S** at **q**.

If not, it is in the normal fan at **q**

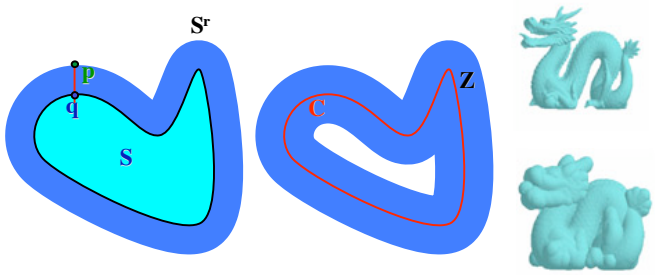


Dilation $\mathbf{S}^r$ and tolerence zone Z

The **dilation** $\mathbf{S}^r$ of **S** = { **p** : $D_{Min}(\mathbf{p},\mathbf{S}) \leq r$ }

= r-offset = Minkowski sum with a ball of radius r

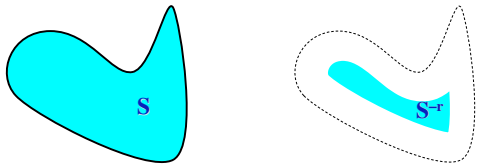
The dilation of a curve **C** defines a **tolerance zone** Z around it



Contraction $\mathbf{S}^{-r}$

The **contraction** of **S**, denoted $\mathbf{S}^{-r}$, is $\mathbf{S} - (b\mathbf{S})^{-r}$

Remove all points within distance r from the boundary b**Q**



Fill F_r and Round R_r

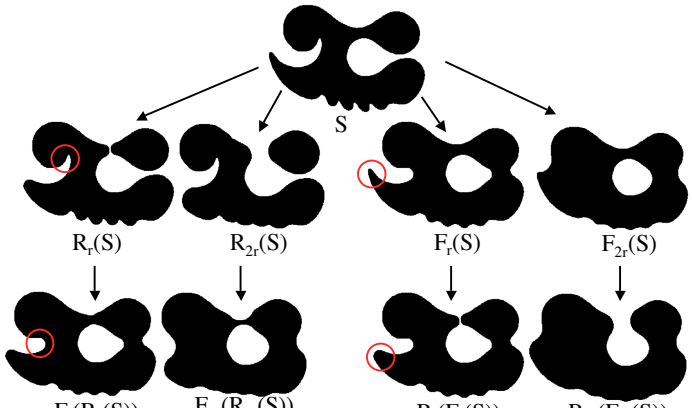
- The r-filleting (or simply **fill**) $F_r(\mathbf{S}) = (\mathbf{S}^r)^{-r}$
  - Morphological closing with a ball of radius r [Serra 82]
  - $(\mathbf{S}\uparrow r)\downarrow r$ [Rossignac 85]

add area not reachable by r-disks out of S

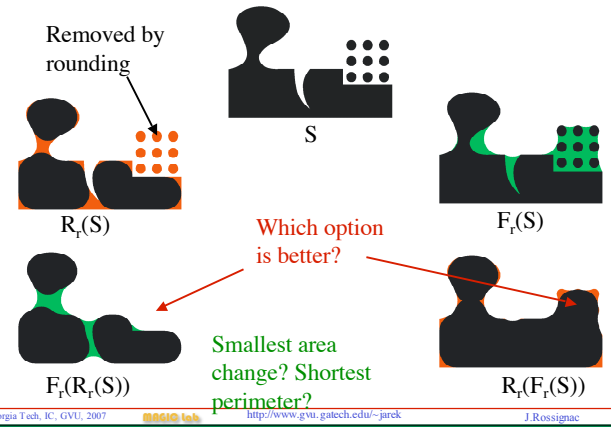
- The r-rounding (or simply **Round**) $R_r(\mathbf{S}) = (\mathbf{S}^r)^{-r}$
  - Morphological opening with a ball of radius r [Serra 82]
  - $(\mathbf{S}\downarrow r)\uparrow r$ [Rossignac 85]

remove area not reachable by r-disks in S

Fill & Round combos



Which is better: FR or RF?



Mortar M_r

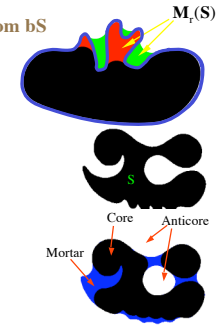
$\mathbf{M_r(S)}$ = **not reachable by open r-balls disjoint from bS**

Molecular surface [Connolly 83]

α-hull(A) for α=1/r [Edelsbrunner 93]

$\mathbf{M_r(S) = F_r(bS)}$

$\mathbf{M_r(S) = F_r(S) - R_r(S)}$

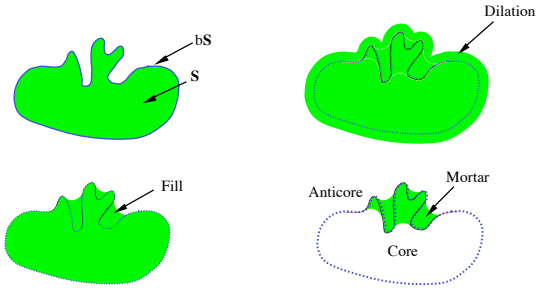


core, $\mathbf{R_r(S)}$
mortar, $\mathbf{M_r(S)}$
aniticore, $\mathbf{!F_r(S)}$

Morphological operators (summary)

r-ball : open ball of radius r

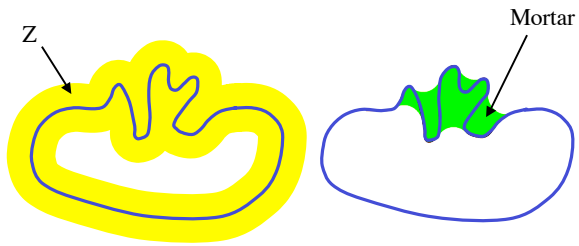
- **Dilation**, $\mathbf{S^r}$: union of r-balls with centers in **S**
- **Fill**, $\mathbf{F_r(S)}$: not reachable by r-balls disjoint from **S**
- **Mortar**, $\mathbf{M_r(S)}$: not reachable by r-balls disjoint from b**S**



Restricting simplification to the mortar

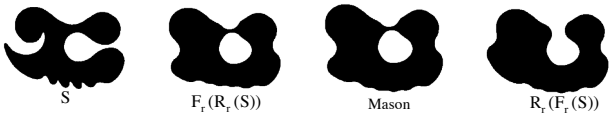
- We want to restrict all changes to a tight zone around the boundary bS of the (solid) shape S
  - Tolerence zone $Z=(bS)^r$
  - Mortar $M_r(S)=((bS)^r)^{-r}$

Simplify in the mortar!
Do not mess up the rest.



The Mason filter

For each connected component M of $M_r(S)$ replace $M \cap S$ by $M \cap F_r(R_r(S))$ or by $M \cap R_r(F_r(S))$, whichever best **preserves the shape**



Preserves density (average area) better than a global $\mathbf{F_r(R_r(S))}$ or $\mathbf{R_r(F_r(S))}$, but does not guarantee smoothness nor minimality of perimeter

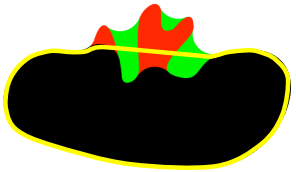
"Mason: Morphological Simplification", J. Williams, J. Rossignac. Graphical Models, 67(4)285:303, 2005.

3D Mason

Tightening is better than Mason

The tightening $\mathbf{T_r(S)}$ may be obtained by tightening b**S** in $\mathbf{M_r(S)}$

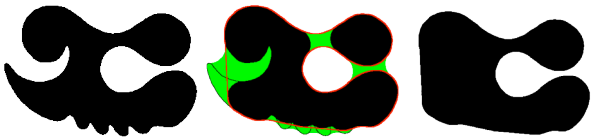
- 2D: The shortest of all *homotopic* curves in $M_r(A)$
- 3D: Minimal area surface in Mortar



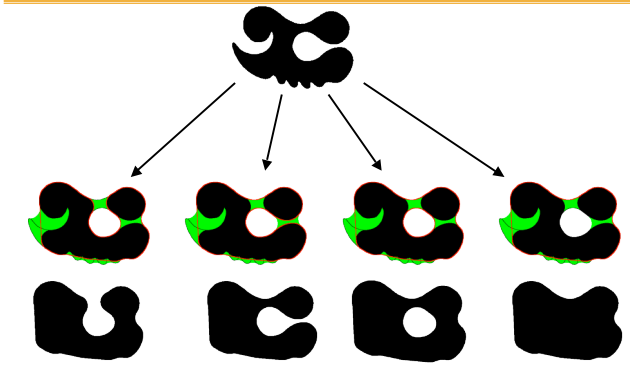
Related to the Tight Hull [Sklansk&Kibler76 , Robinson97, Mitchell04], but the hull is the mortar!

Tightening: Curvature-Limiting Morphological Simplification. J. Williams & J. Rossignac. Sketch in the *ACM Symposium on Solid and Physical Modeling* (SPM). pp. 107-112, June 2005.

Example of tightening



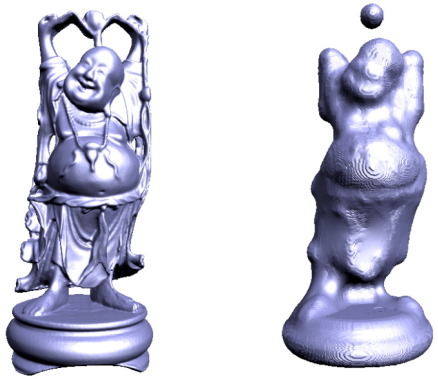
Topological choices of tightening



Different topology

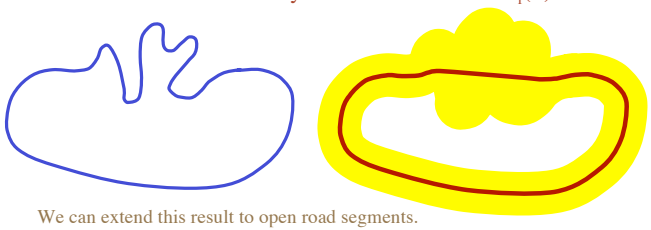
Invalid

3D tightening

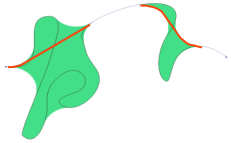
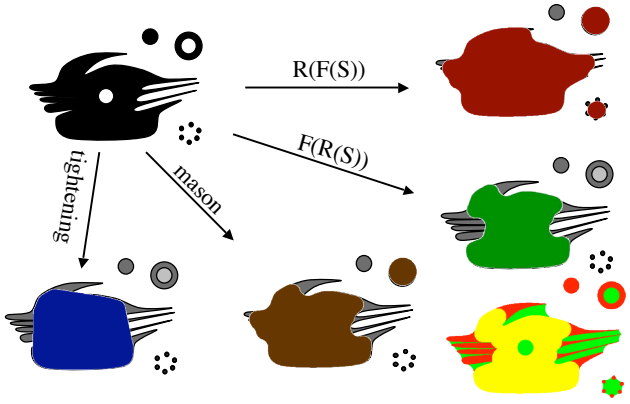


The road-tightening solution

- Let S be the area enclosed by C. The new road is $bT_r(S)$.



We can extend this result to open road segments.



Comparing morphological simplifications