Georgia Tech College of Computing Interactive & Intelligent Computing

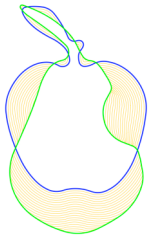
MAGIC lab
Modeling, Animation, Graphic, Interaction & Compression

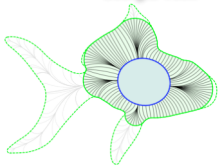
GVU center georgia tech

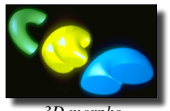
Shape comparison, simplification, & compression: **Discrepancy measures, maps, & morphs**

Jarek Rossignac

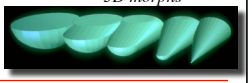
Georgia Tech

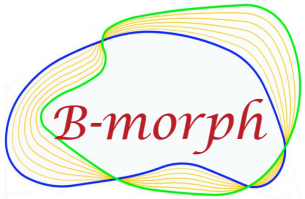






3D morphs





Frederic Chazal, Math. Burgundy & INRIA Futurs

Andre Lieuther, Dassault Systemes & LMC/IMAG

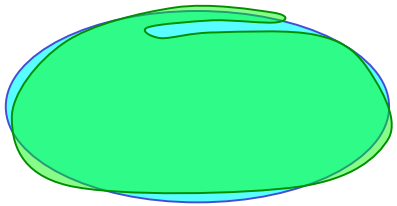
Jarek Rossignac & Brian Whited, Gatech, Atlanta, USA

"OrthoMap: Homeomorphism-guaranteeing normal-projection map between surfaces". F. Chazal, A. Lieutier, and J. Rossignac. ACM Symposium on Solid and Physical Modeling (SPM). pp. 9-14, 2005. GIT-GVU-04-28

"Ball-map: Homeomorphism between compatible surfaces". F. Chazal, A. Lieutier, J. Rossignac, B. Whited. GVU Report GIT-GVU-06-05

Discrepancy Measures

- How can we measure the discrepancy between two shapes?
  - Local measure? If so, at each point of each shape or of space?
  - Global measures? If so, max, average?



Motivation

Dimension and tolerancing of machined parts

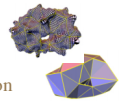
Specify allowable discrepancy in shape

Manufacturing inspection and yield control

Visualize areas where the process deviates from nominal

Bone grafting

Ensure that the graft perfectly matches the desired shape



Shape Simplification

Measure the discrepancy between original and approximation

Animation

Morph between two similar shapes

Rendering

Map textures between different shapes

Analysis

Track/transfer properties between (evolving) (iso) surfaces

Hausdorff distance

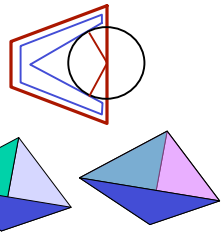
$D_{Haus}(\mathbf{P},\mathbf{Q}) = \min r : \mathbf{Q}^r \subset \mathbf{P}$ && $\mathbf{P}^r \subset \mathbf{Q}$

It is expensive to compute

even in 2D

It is a poor measure of discrepancy

it is orientation and topology insensitive



Parametric and Fréchet distance

Parameterized curves: **P**(t) and **Q**(s) for s and t ∈[0,1]

Map φ: [0,1]→ [0,1]

Parametric discrepancy:

$D_\varphi(\mathbf{P},\mathbf{Q}) = \max \| \mathbf{P}(t) - \mathbf{Q}(\varphi(t)) \| \ : \ t \in [0,1]$

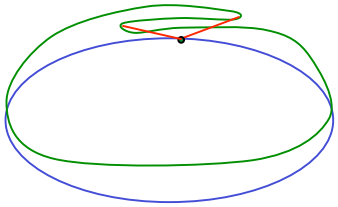
Fréchet distance:

$D_{Fre}(\mathbf{P},\mathbf{Q}) = \min D_\varphi(\mathbf{P},\mathbf{Q}) : \forall \varphi$

Man walks forward on P.

Dog walks forward on Q.

Length of shortest leash?



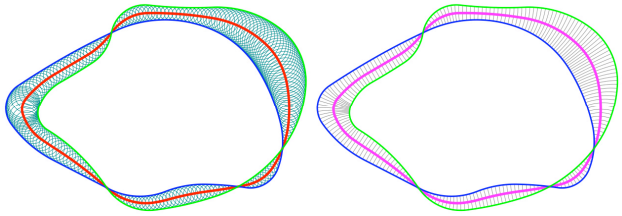
Ball distance ✷new✷

$D_{Ball}(\mathbf{P},\mathbf{Q})$ = Diameter of largest ball that fits in the **gap** and touches both shapes

Good measure if shapes are **b-compatible** (single point contacts)

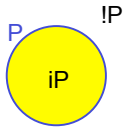
Takes distance and orientation into account.

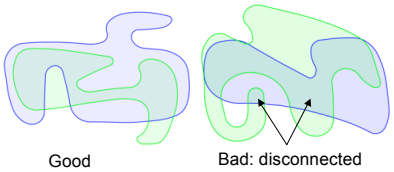
Can be extended to incompatible shapes



Assumption / restrictions

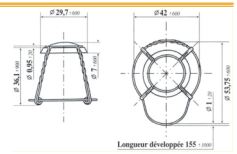
- **P** and **Q** are boundaries (curves in 2D, surfaces in 3D) of regularized regions (area, solid)
  - **iP** is the finite interior (region) bounded by **P**
  - **!P** is the exterior (complement of **P**∪**iP**)
- We assume that:
  - **P** and **Q** are connected and manifold
  - **iP**∩**iQ** is connected
  - **!P**∩**!Q** is connected



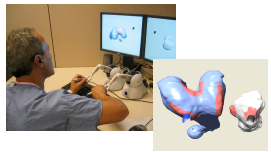


Applications

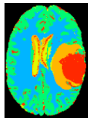
- Compare manufactured shape to nominal design



- Compare shape of weak cartilage to graft



- Compare stages in evolution of tumor or lake

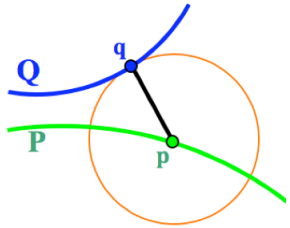


Old "Closest Point" c-map & c-morph

Most of prior work was based on the closest point map or on intrinsic shape segmentation and matching (salient features, skeleton, parameterization…)

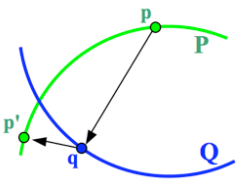
$C_{P\to Q}$ maps **p∈P** to the set of its closest points **q∈Q**

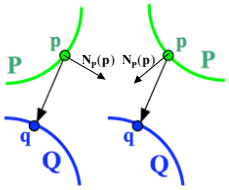
c-morph moves **p** towards **q** at constant speed



C-map drawbacks

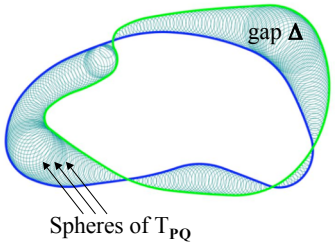
- Not symmetric: $C_{Q\to P}(C_{P\to Q}(\mathbf{p}))\neq\mathbf{p}$
- Insensitive to orientation: same **q**, $\forall \mathbf{N_P}(\mathbf{p})$

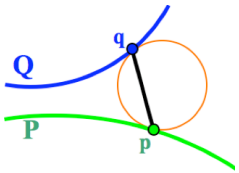




Our "Tangent Ball" **b-map**

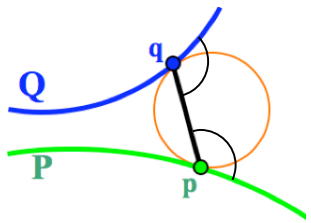
- **Δ**= (**iP⊕iQ**) ∪ (**P∩Q**) is the **gap** between the two shapes
- T_{PQ} is the set of **spheres** in **Δ** that touch both **P** and **Q**
- $B_{P\leftrightarrow Q}$ maps **S∩P** to **S∩Q** for each sphere **S** of T_{PQ}





B-map advantages

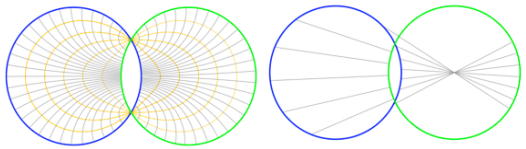
- $B_{\mathbf{P}\leftrightarrow\mathbf{Q}}$ is symmetric: $B_{\mathbf{Q}\leftrightarrow\mathbf{P}}(B_{\mathbf{P}\leftrightarrow\mathbf{Q}}(\mathbf{p}))=\mathbf{p}$
- $B_{\mathbf{P}\leftrightarrow\mathbf{Q}}$ is sensitive to the orientation of both shapes



Comparing maps for compatible pairs

To compare the two, initially assume c- and b-compatibility, even though many pairs that are b-compatible are not c-compatible

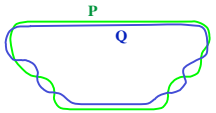
- **P** and **Q** are **c-compatible** when both $C_{\mathbf{P}\to\mathbf{Q}}$ and $C_{\mathbf{Q}\to\mathbf{P}}$ are homeomorphisms
- c-compatibility implies b-compatibility
- hism



Sufficient compatibility conditions

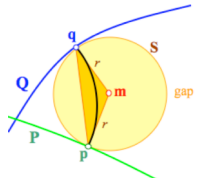
- h = **Hausdorff distance** between **P** and **Q**.
  - h = smallest r such that $\mathbf{P}\subset\mathbf{Q}^r$ and $\mathbf{Q}\subset\mathbf{P}^r$,
  - where $\mathbf{X}^r$ is the set of points at distance r or less from **X**.
- f = min(mfs(**P**) , mfs(**Q**))
  - minimum feature size mfs(**X**) = largest r such that $\mathbf{X}=\mathbf{F}_r(\mathbf{X})$
  - where $\mathbf{F}_r(\mathbf{X})$ is the set not reachable by open balls not intersecting **X**
- $h<f \;\Rightarrow\;$ **P** and **Q** are b-compatible
- $h<(2-\sqrt{2})f \;\Rightarrow\;$ **P** and **Q** are c-compatible

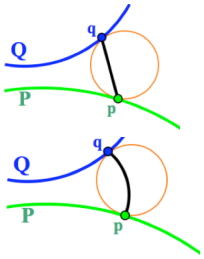
  $2-\sqrt{2} \approx 0.58$

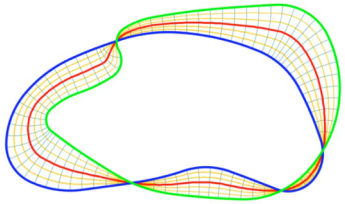


B-morph with circular trajectories

- Circular trajectories
- Normal incidence on both shapes

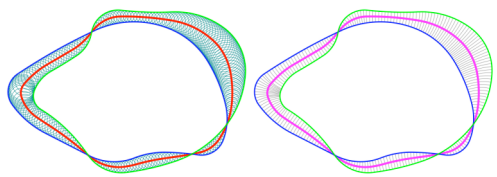






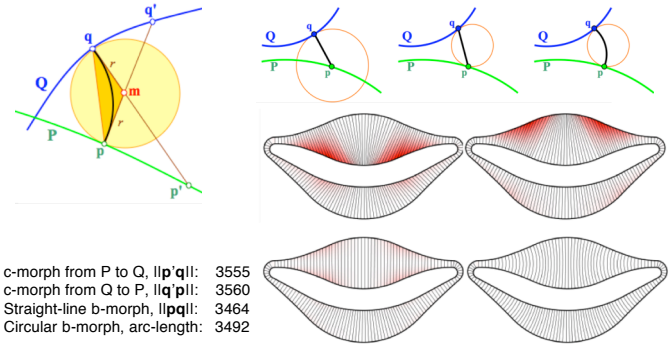
Median M and half-arc curve H

- The median **M** is the set of centers of the spheres of T_{PQ} if $h<f$
  - M has their topology
  - **M** is C^k if P and Q are C^k
- **H** is the locus of mid-arc curves, it is very close to **M**



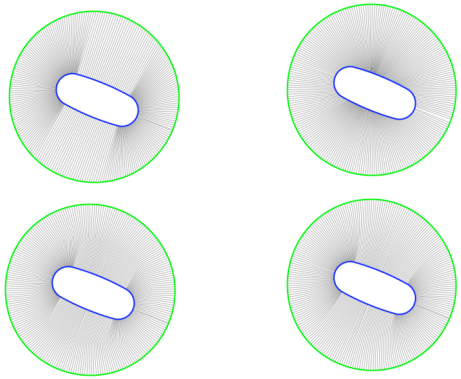
Average travel distance

b-morph requires less average travel than either c-morph when integrated over the median or over both shapes



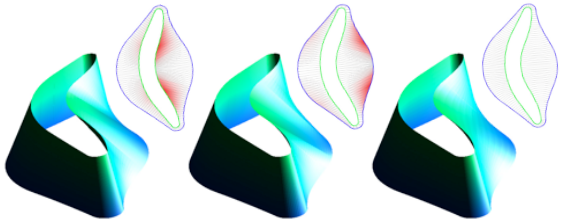
c-morph from P to Q, ||**p'q**||: 3555
c-morph from Q to P, ||**q'p**||: 3560
Straight-line b-morph, ||**pq**||: 3464
Circular b-morph, arc-length: 3492

Average travel distance: case study



Fitting surface between cross-sections

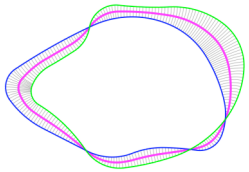
The b-morph yields an interpolating surface with
a smaller surface area
and a "nicer" shape



Two c-morphs and a b-morph. Which one is the "nicest"?

Discrepancy between b-compatible shapes

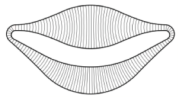
- Local: length of arc from **p**
- Global: Integral of arc-length trajectory
  - Integrate over **M**, **H**, or **P** & **Q**
  - L_1, L_2, L_∞



- When *h<f:*

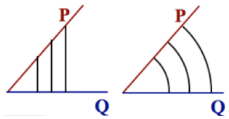
  length of longest arc = $D_{Haus}(\mathbf{P},\mathbf{Q})$

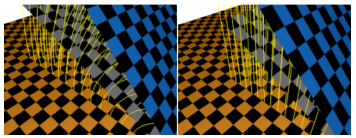
  = $D_{Fre}(\mathbf{P},\mathbf{Q})$



Distortion

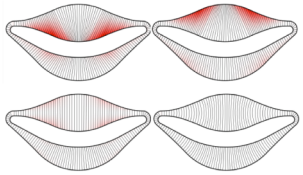
Ball morph between linear sections is distortion free





and in general is closer to normal flow

	$C_{Q\to P}$	$C_{P\to Q}$	Linear $B_{P\leftrightarrow Q}$	Circular $B_{P\leftrightarrow Q}$
$\sum E(s,t)/S(s,t)$	2390	2373	2132	**2118**
$\sum E(s,t)/\sum S(s,t)$	1.0378	1.0370	1.0090	**1.0005**



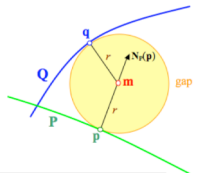
b-map computation

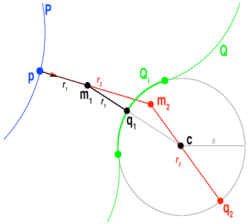
- Grow tangent ball at **p** in the gap until it touches **Q**.

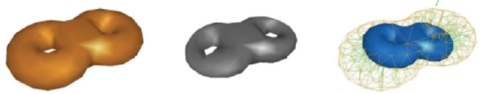
  $\mathbf{m} = \mathbf{p} + r\mathbf{N_P}(\mathbf{p})$ find smallest r such that Dist(**m**,**Q**)==*r*

  Solve for all elements of **Q** (points, line segments, arcs, triangles…).

  Return smallest *r*>0





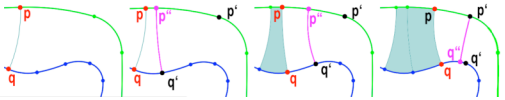


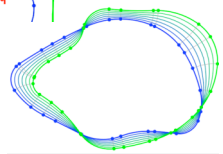
Lacing PCCs

Linear cost algorithm for b-compatible piecewise circular curves

- March on both curves
- Compute the images of the end-points of current edge
- Take the smallst of the two steps

Divides the gap into slabs, each bounded by 4 arcs





B-offset reconstruction

Express **Q** as the b-offset of **P**

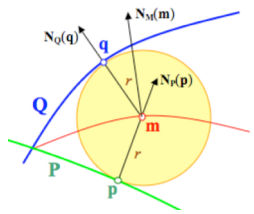
Represent it by **P** and the offset field **r**(p)

Reconstruct **Q** by computing:

$\mathbf{m}=\mathbf{p}+r\mathbf{N}_P(\mathbf{p})$

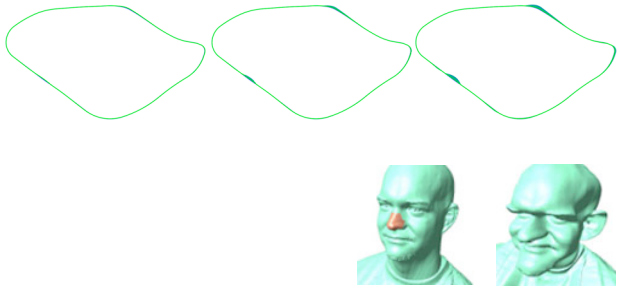
N = normal to **M** at **m** (from derivatives of r in tangent plane)

$\mathbf{q}=\mathbf{p}+2(\mathbf{pm}\bullet\mathbf{N})\mathbf{N}$



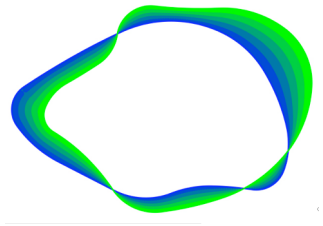
Exaggeration

Barely noticeable discrepancies between b-compatible shapes may be exagerated by scaling the field r



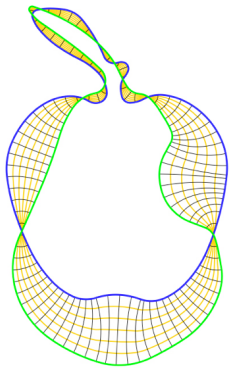
Progressive offset

Increasing the scaling of r from 0 to 1 creates a morph that sweeps the gap



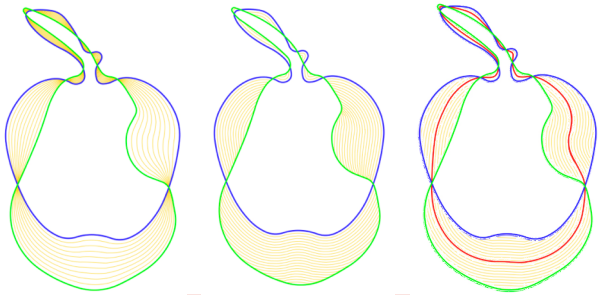
B-morph

- Each sample travels on its circular arc from **p** to **q**



B-morph synchronization

- **Regular**: Constant speed for each trajectory
  - Computed so that they all start and stop at the same time
- **Invading**: Same speed for all, starting at **P**
- **Symmetric**: Same speed for all, simultaneously passing H

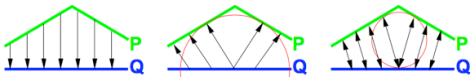


Non b-compatible shapes

Assume now that **P** and **Q** are not b-compatibile

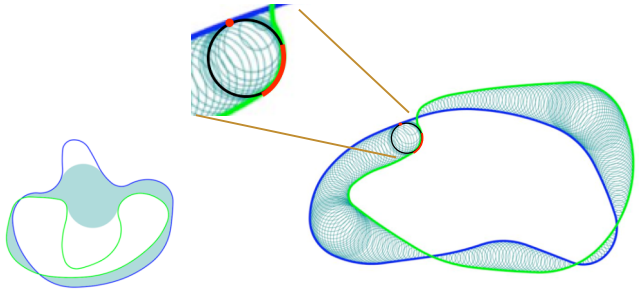
One of the c-maps can still be a homeomorphism

$C_{P\to Q}$ can be a homeomorphism, while $C_{Q\to P}$ and $B_{P\leftrightarrow Q}$ are not



The canal C in the gap Δ

- The **canal** C is the union of all spheres of T_{PQ}
- P and Q are **b-compatible** ⇒ C=Δ
- (C=Δ) && (not b-compatible) ⇒ **quasi-compatible**

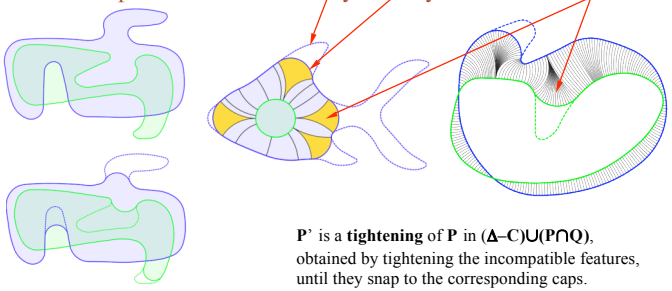


Relative b-roundings **P'** and **Q'**

Modified lacing identifies **incompatible features** that cannot be reached by any sphere of T_{PQ}

It replaces each one of them by a circular **cap**

The b-morph becomes one-to-many or many-to-one in the **fans**



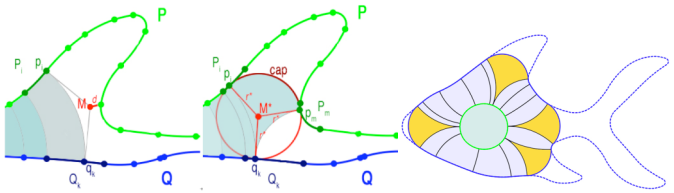
P' is a **tightening** of **P** in **(Δ–C)∪(P∩Q)**, obtained by tightening the incompatible features, until they snap to the corresponding caps.

b-rounding via modified lacing

if Dist(**m**,**Q**)<r, find center **m*** and radius **r*** of a circle **K** in Δ that is tangent to $\mathbf{P}_i$, to $\mathbf{Q}_k$, and to another edge $\mathbf{P}_m$ of **P**

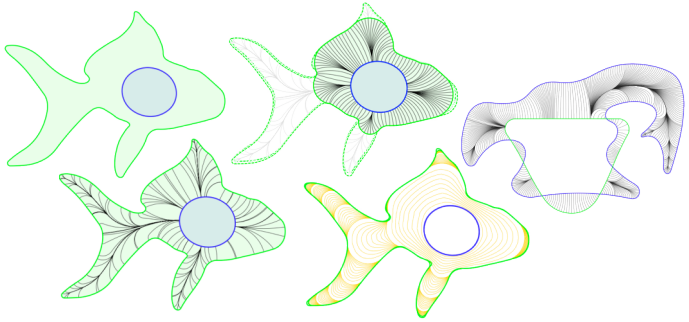
(the first one found along P)

The portion of **K** between $\mathbf{p}_i$ and $\mathbf{p}_m$ is the **cap** and replaces the corresponding incompatible portion of **P**.

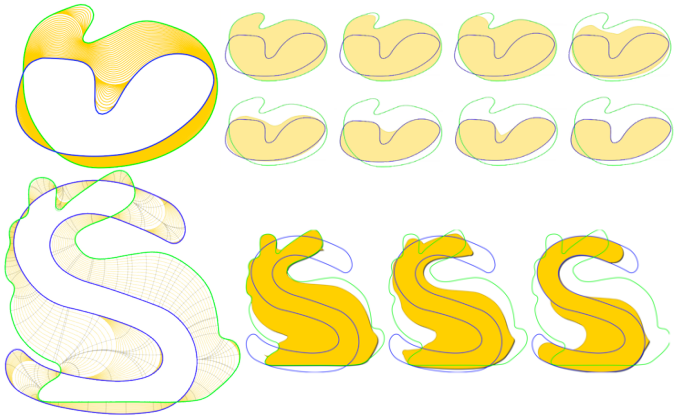


Composite b-morph

- Compute b-roundings $\mathbf{P}_1$ =bRound(**P**,**Q**), $\mathbf{Q}_1$=bRound(**Q**,**P**)
- Then recursively: $\mathbf{P}_{i+1}$=bRound(**P**,$\mathbf{P}_i$), $\mathbf{Q}_{i+1}$=bRound(**Q**,$\mathbf{Q}_{i+1}$)
- Stop when **P**=$\mathbf{P}_i$ or when $C_{\mathbf{P}\rightarrow \mathbf{p}_i}$ is a homeomorphism
- Note that trajectories are smoothly connected PCCs

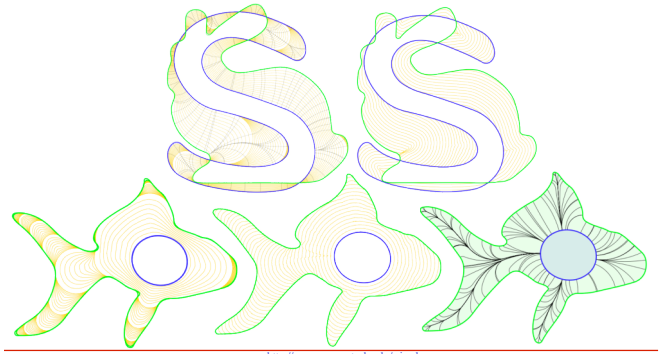


Examples of composite b-morph



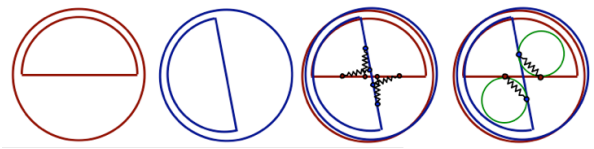
Synchronization of composite b-morphs

Regular: Each morph has constant speed
Invading: All same speed staring at one shape
Symmetric: Same speed, synchronized at H



Registration

- The b-map is better than the c-map for ICP registration



Summary

- B-map and b-morph are better than c-map and c-morph
  - symmetric
  - orientation sensitive
  - shorter travel and smaller surface area
  - less (or no) distortion
  - more compatible (except near sharp features)
  - implement Frechet distance (if h<f)
- We have defined then for b-compatible sets
- We have implemented them for PCCs and T-meshes
- We have extended them to incompatible PCCs