Multiresolution Wavelet Based Model for Large Irregular Volume Data Sets

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ABSTRACT

In this paper we propose a wavelet based model for large irregular volume data sets by exploiting a multiresolution model based on semi-regular tetrahedral meshes. In order to generate the multiresolution representation we use a wavelet based approach that allows compression and progressive transmission. Beginning with a semi-regular tetrahedral mesh $\Gamma_\infty(T)$ and applying the wavelet transform, we obtain a representation that consists of a coarse base mesh $\Gamma_0(T)$ and a sequence of detail coefficients obtained from the sucessive decomposition of the mesh at different levels of resolution. The base mesh is the one at the lowest resolution and it does not have the connectivity subdivision property. The wavelet decomposition is obtained by defining a wavelet basis over tetrahedra generated by a regular subdivision method applied to an initial tetrahedron T. The obtained basis is a Haar-like one and forms an unconditional basis for $L^p(T,\Sigma,\mu)$, $1<p<\infty$, being μ the Lebesgue measure and Σ the σ-algebra generated from the tetrahedron T by the chosen subdivision method.

Keywords

Volume Modeling, Multiresolution Modeling with Wavelets, Irregular Volume Modeling, Wavelets

1. INTRODUCTION

Three dimensional scenes contain highly detailed geometric models for many applications such as Internet 3D models for complex virtual environments, collaborative CAD, interactive visualization, etc. This situation motivates the development of 3D surface and volume models in order to meet requirements like effective use of disk space and network bandwidth, as well as substantial reduction of network transfer time.

Multiresolution representations have become a key technology for achieving efficient storage together with progressive transmission and visualization performance. Besides that, wavelet-based methods have proven their efficiency for the visualization at diferent levels of detail, progressive transmission and compression of large data sets.

A volume data set is often modeled by a mesh consisting of tetrahedral cells with scalar and/or vector data associated to them. In computer graphics, much research has been devoted to nested tetrahedral meshes generated by recursive decomposition, which are suitable to deal with regularly distributed data points. On the other hand, multiresolution models based on irregular tetrahedral meshes are desirable in order to deal with irregularly-distributed data, since they can accurately capture the shape of the field domain even at the lowest resolution. In this work, we propose an effective 3D model scheme for large volume data that exploits the power of wavelet theory. To achieve this, we present wavelets on tetrahedral domains and we use them as a multiresolution approach that can deal with irregular volume meshes that satisfies the subdivision-connectivity property. That is, we assume that the mesh on which the data is defined can be reached by

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recursive subdivision of a base mesh and the volumetric data set in encoded as a base mesh (coarse approximation) followed by a sequence of detail coefficients.

The paper is organized as follows: in Section 2 we present an overview of related work on wavelets applied to object modeling. In Section 3 we define Haar-like wavelets over a tetrahedron, we give the filter coefficientes for analysis and synthesis, and we develop an example where a density function defined on a tetrahedron is represented using these waveletes. In Section 4 we present the detailed explanation of wavelet based modeling for irregular volumes, we provide the data structure and we justify why this model allows compression and progressive transmission. Finally, in Section 5 we draw some conclusions and we give an outlook over future work.

2. RELATED WORK

Different methods have been formulated for modeling volumes using tensor products ([Mur00], [Chu00]). The idea of using three dimensional wavelets to approximate three-dimensional volume data sets was introduced in [Mur00], where he constructs a 3D orthonormal wavelet basis using all possible tensor products of one-dimensional basis functions and presents the potential of the 3D wavelet transform (WT) for volume visualization. Although this methodology gives a simple way for constructing wavelets for surface or volume representation; it cannot be used without introducing degeneracies when representing surfaces or volumes defined on general domains of arbitrary topological type, like spherical domains.

Lounsbery [Lou00] and Stollnitz *et al.* [Sto00] were the first who introduced wavelets from a different point of view, defining them on different bounded domains through scaling refinable functions. Refinability, a key property for multiresolution analysis (hereafter referred to as MRA), generalizes the notion of both translation and dilation. Within this context, wavelets defined on arbitrary topological domains on $\Re^2$ are built up in [Lou00]. This approach was generalized *a posteriori* by Sweldens ([Swe00], [Swe01]) who recognized that the *lifting scheme* he proposed was a generalization of Lounsbery's methodology. Other subdivision wavelet construction for functions defined on triangulated spherical domains were introduced by Schröder and Sweldens [Sch00], Nielson *et al.* [Nie00], Bertram *et al.* [Ber00], and Bonneau [Bon00].

In [Lab00] and [Lin00], other techniques for representing volume data are given. However, a tetrahedral mesh can be used to model an object given by sparse data, and this mesh is a general topological domain for the intrinsic representation of the volume. In this case, it is necessary to have wavelets defined over tetrahedra and, based on sucessive refinements, extend the multiresolution analysis to functions defined on them. Hence, beginning with a tetrahedron and using the subdivision as a construction tool, it is possible to generate wavelets on arbitrary topological domains of dimension three or greater. In this paper we present a Haar-like wavelet basis defined over a tetrahedron as the first step for defining this kind of bases over an object represented by tetrahedra.

3. WAVELET BASIS DEFINED OVER A TETRAHEDRON

In order to define the wavelets, it is necessary to adopt a subdivision method. For 2D triangular meshes there are refinement methods satisfying nestedness, consistency and stability. Perhaps the most known one is the combination of red and green refinements proposed by Bank *et al.*, [Ban00]. The stable refinement of tetrahedral meshes is more complex; there is no way a given tetrahedron can be divided into eight congruent ones. However, it is possible to extend the regular 2D refinement strategies mentioned above to 3D. For this purpose different algorithms have been developed during the last years (see, for example, [Bae00], [Mau00], [Bey00]).

Bey [Bey00] introduced a refinement algorithm for unstructured tetrahedral grids, which generates stable and consistent triangulations. In order to do this, he defined some local regular and irregular refinement rules that are applied to single elements. Since our multiresolution model is a regular nested model [Mag00] based on the recursive subdivision of tetrahedra, we choose this method because it is highlighted by the fact that any given tetrahedron results in a group of elements of at most three congruence classes, no matter how many successive refinement steps are performed.

Beginning with a tetrahedral net and using this subdivision method, it is possible to construct wavelets on arbitrary topological domains. To achieve this goal, it is first necessary to define them on a single tetrahedron. In this work we concentrate ourselves on this problem and we provide an example of a L^2 functions representation on the defined bases, as well.

3.1 Haar-like Wavelets

In this section we build a wavelet basis on a tetrahedron following the process given by Girardi and Sweldens [Gir00]. Throughout this section, (*T,*

Σ, μ) is the measure space where T is a tetrahedron with volume V , Σ is the $\sigma-$algebra of all tetrahedra generated by the Bey's subdivision method and μ is the Lebesgue measure. With this procedure we shall obtain Haar-like wavelets on the measure space (T,Σ, μ) that form an unconditional basis for $Lp(T, \Sigma, \mu)$, $1 < p < \infty$.

3.1.1 Wavelets Construction

According to the chosen subdivision scheme, eight new tetrahedra $T_{\beta i}$, $1\leq i \leq 8$, obtained by subdivision of the father tetrahedron T_α, are introduced during synthesis. This means that these tetrahedra will be generated when going from a resolution level j to a finer resolution level $j + 1$ and will replace the coarser resolution tetrahedron T_α. [Cas00]

Then the refinement equation can be written:

$$\varphi_\alpha = \sum_{\beta \in B(\alpha)} \frac{1}{\sqrt{8}} \varphi_\beta ,$$

being $B(\alpha)$ the set of tetrahedra obtained after subdividing the tetrahedron T_α .

The two scale equation for the wavelets is:

$$\psi_\gamma = \sum_\beta g_{\gamma,\beta} \varphi_\beta ,$$

being $g_{\gamma,\beta} = \langle \psi_\gamma , \varphi_\beta \rangle$ and $\gamma \in G(\beta)$, $G(\beta)$ the detail tetrahedra obtained when going from resolution $j + 1$ to resolution j.

Hence, a single step in the analysis is given by:

$$\varphi_\beta = \frac{1}{\sqrt{8}} \varphi_\alpha(\beta) + \sum_{\gamma \in G(\beta)} g_{\gamma,\beta} \psi_\beta .$$

The generated multiresolution spaces are:

$$V_{k+1} = closspan\{\varphi_\beta , \beta \in B(\alpha)\}$$
$$W_{k+1} = closspan\{\psi_\gamma , \gamma \in G(\beta)\}$$

being

$$V_{k+1} = V_k \oplus W_k .$$

Then, a given function $f \in V_{k+1}$ has two representations:

$$f = \sum_{\beta \in B^*(\alpha)} c_\beta \varphi_\beta ,$$

with $B^*(\alpha)$ the set of tetrahedra at the highest resolution and

$$f = \sum_{\alpha \in F_j} c_\alpha \varphi_\alpha + \sum_{\gamma \in G_j} d_\gamma \psi_\gamma ,$$

being F_j the set of tetrahedral at lower resolution levels and G_j the set of details obtained from the decomposition of the mesh until a given level j. As always, c_α and d_γ are found with the fast Haar WT applied to the original signal. (see Sections 3.1.1.1). Then, we must compute these coefficients during the process of analysis and synthesis.

3.1.1.1 Fast Wavelet transform

Analysis

We obtain the low resolution coefficient $c_{j,\alpha}$ from $c_{j+1,\beta}$ with

$$c_{j,\alpha} = \sum_{\forall \beta \in C_j(\alpha)} \frac{1}{\sqrt{8}} c_{j+1,\beta} ,$$

while the details coefficients are:

$$d_{j,\gamma} = \sum_{\beta \in C_j(\alpha)} g_{\gamma,\beta} c_{j+1,\beta} ,$$

where:

$$d_{j,\gamma_1} = \frac{1}{\sqrt{8}} \left(c_{j+1,\beta_1} + c_{j+1,\beta_2} + c_{j+1,\beta_3} + c_{j+1,\beta_4} \right) - \frac{1}{\sqrt{8}} \left(c_{j+1,\beta_5} + c_{j+1,\beta_6} + c_{j+1,\beta_7} + c_{j+1,\beta_8} \right)$$

$$d_{j,\gamma_2} = \frac{1}{2} \left(c_{j+1,\beta_1} + c_{j+1,\beta_2} \right) - \frac{1}{2} \left(c_{j+1,\beta_3} + c_{j+1,\beta_4} \right)$$

$$d_{j,\gamma_3} = \frac{1}{2} \left(c_{j+1,\beta_5} + c_{j+1,\beta_6} \right) - \frac{1}{2} \left(c_{j+1,\beta_7} + c_{j+1,\beta_8} \right)$$

$$d_{j,\gamma_4} = \frac{1}{\sqrt{2}} \left(c_{j+1,\beta_1} - c_{j+1,\beta_2} \right)$$

$$d_{j,\gamma_5} = \frac{1}{\sqrt{2}} \left(c_{j+1,\beta_3} - c_{j+1,\beta_4} \right)$$

$$d_{j,\gamma_6} = \frac{1}{\sqrt{2}} \left(c_{j+1,\beta_5} - c_{j+1,\beta_6} \right)$$

$$d_{j,\gamma_7} = \frac{1}{\sqrt{2}} \left(c_{j+1,\beta_7} - c_{j+1,\beta_8} \right)$$

Synthesis

To recover $c_{j+1,\beta}$ from $c_{j,\alpha}$ and $d_{j,\beta}$, we use:

$$c_{j+1,\beta} := \frac{1}{\sqrt{8}} c_{j,\alpha} + \sum_{\gamma \in G_j(\beta)} g_{\gamma,\beta} d_{j,\lambda} .$$

As the obtained wavelets are orthogonal, the coefficients $g_{\gamma,\beta}$ for the synthesis are equal to those ones used for the analysis.

3.1.1.2 Funtions defined on volumes

We can represent scalar or vector functions defined on volumes using the wavelets defined on a tetrahedron. A scalar function can represent, for example, a density, a temperature, etc. As usual, the fast wavelet transform can be directly applied to the set of coefficients that represents the scalar function. If this function has been mapped to color or texture, the wavelet transform is applied to the mapped function. In Figure 1, we show two steps in the analysis. In this case, the attribute has been mapped to color and the wavelet decomposition has been performed to it.

Figure 1. Wavelet Decomposition of an Attribute defined on a Tetrahedron

4. VOLUME MODELING USING WAVELTS

The mesh representing the object must store the 3D geometry, its topology and its attributes. One of the main advantages of tetrahedral meshes is that any other polyhedral mesh can be reduced to a tetrahedral mesh; hence a tetrahedral mesh can represent a volume with arbitrary topological type. Then, beginning with a tetrahedral mesh and using the subdivision and the defined wavelets, we will show how to generate a model of a volumetric object of arbitrary topological type that can be used for compression and progressive transmission.

4.1 Volumetric Model

The proposed model is based on a semi-regular mesh, which is regular except on the coarsest level tetraheda. This kind of mesh is especially well suited for different multiresolution algorithms. A semi-regular tetrahedral representation is a sequence of approximations at different resolution levels. The corresponding sequence of nested refinements is transformed using the wavelet transform to a representation that consists of a coarse resolution or base mesh and a set of detail coefficients that represents the differences between succesive resolution levels. The base mesh Γ_0 is the mesh at the coarsest resolution and does not have the subdivision-connectivity property. Then, the model consists of a base mesh and a sequence of modifications. These modifications correspond to terms that locally capture the details of the object at different resolutions.

The construction of the multiresolution model starts with a fine tetrahedral mesh Γ_∞ (Figure 2) with the subdivision connectivity property.

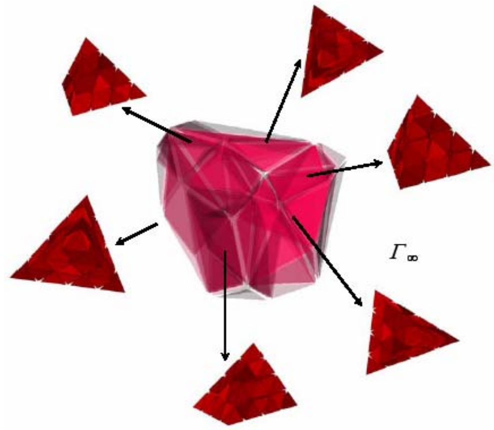

Figure 2. Tetrahedral Mesh

After performing the wavelet transform as many times as possible, we obtain the coarsest resolution mesh Γ_0 plus a set of detail coefficients (Figure 3).

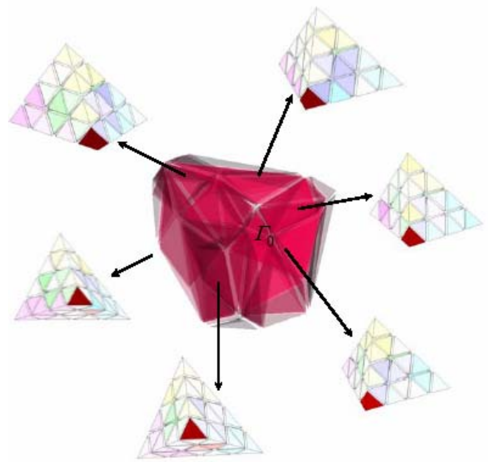

Figure 3. Multiresolution Representation of the Volume

Hence, the developed model begins with the finest resolution mesh Γ_∞ and decomposes it on the coarsest mesh Γ_0, with a set of detail tetrahedra generated during the analysis. This base mesh, together with all the details, are the multiresolution representation of the volume (Figure 3).

Taking into account that the wavelet transform concentrates the energy on the coarsest resolution mesh and that the mesh has space localization, this model is suited for compression. However, the compression will depend not only on the chosen wavelts but also on the following issues:

- The number of coefficients needed to achieve a good approximation of the volume.
- The mesh encoding and storage with the minimun number of bits.

If we transmit the base mesh and all the details, the compression method can be comonsidered as a losless compression encoding. If not all the coefficients are stored, it can be considered a lossy compression encoding. As the highest energy concentration is achieved in the low resolution mesh, only between the 10% and 15% of the detail coefficients are required in order to have a good approximation of the volume

To transmit the underlying mesh of this model *via* Internet, we must consider the transmission of:

- **The base mesh.** A robust transmission has to guarantee that the base mesh is completely transmitted before the details are transmitted and added.
- **The detail coefficients**. After transmitting the base mesh, the details must be sent and added to it.

The transmitted details can be added to the base mesh all at once after they have been received or one by one as long as they are received, until certain requirements are fullfilled. This last option allows the progressive transmission of the model.

4.2 Data Structure

We describe now the data structure proposed for representing the multiresolution volume model. In general, multiresolution meshes admite very compact data structures [Mag00] because the modifications, the involved cells, and the partial order are implicitly defined on the basis of fixed patterns. Then, the data structure for our model must encode the base mesh and the set of details.

The data structure proposed for our model consists on the data structure for the base mesh Γ_0=<σ_0 , σ_1 ,..., σ_n> and on a forest of octrees to store the details corresponding to each one of its cells. Figure 4 shows a tetrahedron of the base mesh and its associated details represented by a forest such its trees have the $d_{0,j}$ as roots. Each tree in the forest is a hierarchy of regular tetrahedra and the relation of dependency is structured as an octree of tetrahedra. As every cell vertex define a regular grid, the coordinates of each vertex can be retrieved from its position in the grid.

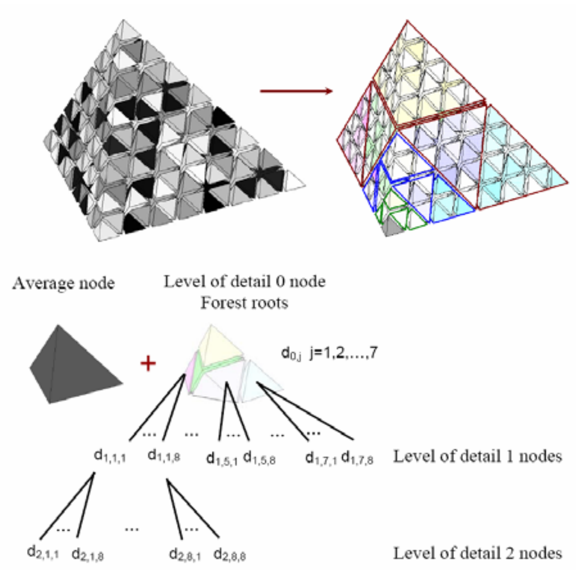

Figure 4. Wavelet Decomposition of a Tetrahedron and graphic Representation of that Decomposition (base mesh+forest)

This multiresolution data structure is generated from a given volume with the conectivity-subdivision property Γ_J, being J the maximum resolution level. The wavelet transform is performed on each set of tetrahedra that replaces the tetrahderon Γ_0 until the lowest resolution tetrahedra Γ_0 is obtained. The set σ_0 coincide with a cell of Γ_0 and its forest of details (Figure 5).

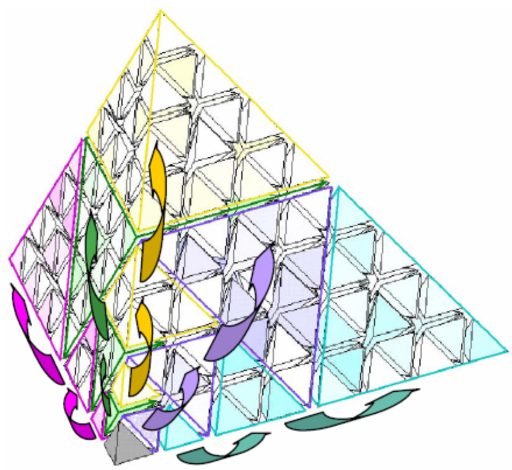

Figure 5. Base Mesh Tetrahedron and the Forest of Details

A forest, the decomposition level and a key to a reference coordinate corresponding to the lowest resolution tetrahedron are stored in a heap for each cell of the base mesh. The reference coordinate will be used to retrieve the geometry of the tetrahedra corresponding to those ones obtained from it at a finer resolution.

4.1.1 Space Complexity of the Data Structure

It is supposed that the mesh is stored using an indexed structure like winged edge, that encodes, for each tetrahedron, the indices of its vertices and the adjacent tetrahedra, along the four faces. The total storage cost corresponding to the data structure of the reference mesh can be calculated in the follong way:

$TotalCost = ConnectivityCost + GeometryCost + AttributesCost.$

If n is the number of vertices of the reference mesh and t is the number of tetrahedra, the amount of t tetrahedra is about 6;, the connectivity cost requires to store $4t$ indices (one for each vertex), $4t$ indices corresponding to the adjacent tetrahedra and $3n$ vertex coordinates. Since the cost of an scalar attribute is t, if we consider that we have only one scalar attribute, the storage cost is:

$TotalCost = 8t + 3n + t = 48n + 3n + 6n.$

Asuming 4 bytes for the indices, 2 bytes for each coordinate and 2 bytes for a scalar attribute, the storage cost in bytes is $210n$ *bytes.* From this, it is clear that the connectivity information dominates the storage cost and it must be compressed.

The storage cost of our model is:

$TotalCost = BaseMesh\ Cost + F\ orestCost.$

The storage cost of the base mesh is the storage cost of a winged edge data structure. So,

$BaseMesh\ Cost = 210n_{bm}$ *bytes*,

being n_{bm} ($n_{bm} << n$) the amount of vertices of the base mesh. Then:

$TotalCost= 210n_{bm}$ *bytes* $+ F\ orestCost.$

Each tetrahedron of the base mesh has a forest associated to it and any other node describes a detail tetraedron; the eight sons corresponds to the tetrahedra obtained from Bey's subdivision method.

Each one of the seven trees of the forest is a complete octree that we can implicitly codify; *i.e.* we do not need to store the conectivity and the structural information; since we have regular tetrahedra, the vertex coordinates are implicit. As a consequence, we only need to encode the field or the attribute values in order to encode the details.

We have supposed that each attribute is stored in 2 bytes. These attributes are stored for each tetrahedron of the base mesh and these values have been taken into account in the storage space required for the base mesh. The number of detail tetrahedra plus the tetrahedra of the base mesh is the number of tetrahedra of the reference mesh; however, for each of them we only store the detail that corresponds to the attribute. Then we have $t - t_{bm}$ detail tetrahedra and the storage cost becomes:

$TotalCost = 204n_{bm}$ *bytes* $+ t - t_{bm}$

$= 210n_{bm}$ *bytes* $+ (6n - 6n_{bm})$*byte.*

The storage cost is significantly reduced compared to the total cost of the reference mesh ($n_{bm} << n$).

4.3 Example

It is worth to notice that when the size of the base mesh compared to the size of the highest resolution one gets smaller, the storage cost diminishes. Table 1 shows a comparison of the storage cost between the reference mesh and our approach at n different resolutions.

	Cost (in bytes)			
n	Reference Mesh	Base Mesh	Forest	Our approach
8	603979776	75497472	14680064	90177536
7	75497472	9437184	1835008	11272192
6	9437184	1179648	229376	1409024
5	1179648	147456	28672	176128
4	147456	18432	3584	22016
3	18432	2304	448	2752
2	2304	288	56	344
1	288	36	7	43

Table 1

5. MESH COMPRESSION AND PROGRESSIVE TRANSMISSION

We give now the structure to represent the complete multiresolution mesh. However, this wavelet based representation can be compressed even more in orden to reduce storage space and transmision time. We will see what elements must be introduced in the structure to achieve higher levels of compresion. Besides, as we want embedded encoding for progressive transmission, we will see how to include it in the structure.

5.1 How the Model allows Compression

We will apply the compression technique to a scalar function defined on the tetrahedra and we will show how the cell based scheme allows to achieve a high compression level. We can have two different alternatives to compress the volume: one is to reduce the number of coefficients to approximate the volumetric data and the other is to encode and to store the necessary information using a small number of bits.

5.1.1 Two models for Compression

We suppose to have the multiresolution model as described in the above sections and that the WT was performed to the given data. Afterwards, the attribute values associated to the tetrahedra are decorrelated and the energy of the original data is concentrated in a relative small number of coefficients.

The key behind wavelet compression is to select the coefficients with smallest norm and replace them by zero. This criterion minimizes the L^2 norm of the resulting approximation error. Whatever is the selected criteria to set the detail coefficients to zero, the original signal will be approximated with a very small number of nonzero coefficients. Then, we can obtain compression in two different ways:

- All coefficients that remain in the representation are encoded with a lower amount of bits per coefficient using run-length encoding, vectorial quantization or differential encoding.
- A very small portion of coefficients (between 10% and 15%, for example) are kept and the rest of them are set to zero. Hence, it would be reasonable to keep only the non zero coefficients. In order to do this, we can take advantage of the spatial localization property of the wavelets: the behaviour of the detail coefficients of a father in a given forest tree allows to predict the behaviour of its descendents.

5.1.2 Error Control

In the models previously described we have supposed that we could control the level of compression by specifying a given percentage of coefficents that are not set to zero (surviving coefficients). In both cases, we can control the number of surviving coeffients by specifying an adequate treshold and set to zero all coefficients whose magnitudes are smaller than it. This threshold can be automatically determined taking into account the maximum allowed approximation error. The ideal way to compute the threshold is by sorting all the coefficients in decresing order of significance. However, when the amount of data is huge, this is impractical (*O(n log n)*). Then, if we want to control the error, we should find the threshold without sorting the data.

One can also specify a threshold and encode all the coefficients of magnitude greater than it and eliminate all the other ones (*O(n)*). In this case, even if the approximation error can be computed, it can not be controled since a fixed number of coefficients are eliminated, depending on their value respect to the threshold.

Finally, the compression scheme developed so far allows compression of non structured volumes decomposed in atomic tetrahedral elements and that have scalar or vectorial values defined on them. It is then necessary to consider the appropiate metric depending on the nature of data, *e.g.* geometric, color, texture data, etc. in general the L^2-norm is considered.

5.1.3 Decompression

The decompression allows to reconstruct the received information of the progressive transmission. The base mesh will be received first and will be decoded according to the encoding method. Once the reconstruction of the base mesh is completed, the inverse WT will be applied to the detail coefficients received afterwards.

5.2 How the Model allows Progressive Transmission

For the mesh transmision we use a modification of the protocol for the transmission of semi-regular meshes, the so-called *mesh transfer protocol (MTP)* defined by Staadt [Sta00]. In order to guarantee a reliable and ordered delivery of the base mesh to destination, we use TCP for the transmission. To do this, the protocol must use a lot of overhead communication; fortunately the base mesh is small compared to the finest resolution level of the mesh.

After the transmission of the base mesh has been completed, the details must be sent. In this case, the protocol used to send them will depend on the implemented model:

- If the details are sent without position information it means that it is implicitely given by the order the chain is transmitted and TCP must be used for detail transmission.
- If each detail record contains the complete topological information that specifies the tetrahedron to which the detail has to be added, the ordering of the detail records is not important. For the reconstruction of the original mesh it is even not necessary that every detail record is received: the loss of records may result only in small local errors. Therefore, it is possible to use the UDP protocol for their transmission.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have extended the definition of the wavelets defined on a single tetrahedron to tetrahedral preserving the MRA. Based on them it was shown that it is possible to represent irregular tetrahedral meshes using a multiresolution approach. We have also considered only scalar functions defined on tetrahedra since they represent the volume attributes. The proposed model allows compression and progressive transmission. In compression, we showed how to reduce the number of coeffients needed for approximating the volumetric data and how to encode the information according to the proposed models. In the case of transmission, we analyzed a protocol that allows the progressive transmission of the mesh.

We consider that the described framework is an important initial work to construct multiresolution representations of irregular meshes. Future work includes the extension of our results to functions defined on unstructured tetrahedral domains and also the representation of the underlying domain geometries. These extensions would allow to obtain a wavelet-based method to model irregular tetrahedral meshes without the subdivision connectivity property. The used oracles and reconstruction schemes are based on the L^2 norm. Furthermore, we are studying the possibility of taking into account other error criteria based on human perception in order to optimize the approximation quality.

7. ACKNOWLEDGMENTS

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