Combining Geometrical and Mechanical Displacements for Suppressing Intersections in Cartographic Generalization

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ABSTRACT

This paper describes a method for suppressing spatial conflicts in cartographic generalization of isobathymetric lines. These lines are modeled by parametric curves. For that purpose, we review our preliminary works on the detection of intersections and examine the current methods used for line displacement. We focus on geometrical methods and on a cable network based method and propose a new strategy. Our algorithm first determines a geometrical solution and then refines it by applying a mechanical deformation.

Keywords

Intersection, displacement, B-spline curve, cable network, line cartographic generalization.

1. INTRODUCTION

This paper deals with the problem of cartographic generalization of isocontour lines in a maritime context (i.e. isobathymetric lines). Cartographic generalization includes the whole processing encountered when the scale of a chart is changed into a smaller scale. A chart can be defined as an abstracted model representing geometric reality. The smaller the scale, the more schematic the representation. Thus, the goal of generalization is to produce charts which are as close as possible to reality while respecting their legibility and the application constraints (see Figure 1). This can be obtained by applying cartographic generalization operators such as selection/elimination, aggregation, structuring, compression (or filtering), smoothing, exaggeration, caricaturing, enlargement and displacement [Rua93].

The interest of using continuous curves such as B-spline curves for modeling isobathymetric lines as

well as for processing some generalization operators has been proved in [Sau03]. Spatial conflicts or intersections can be encountered during the computation of the initial parametric curves (e.g., due to a bad parameterization) or during the application of some generalization operators (e.g., a smoothing process). They have to be detected and located before suppression by local or global displacements. This paper focuses on the problem of suppressing spatial conflicts.

Maritime charts should legally ensure navigation safety of users (and sailors). Consequently, line displacement is characterized by strong constraints. Extending the usual constraint classification proposed by Beard [Bea91], we identify:

- *The graphic constraint of legibility*: The final chart should not include either *real line intersections* (i.e., transversal intersections, overlaps, tangencies) or *visual line intersections* (when curve segments are too close with respect of an accuracy criterion ε_{vis} linked to the working scale)

- *The application constraint of safety*: The displacement of an isobathymetric line ought to be done towards deeper areas to preserve the safety of navigation. More precisely, one can displace a 4m (under the sea level) isobathymetric line to a deeper area (e.g., a

5m area) on a chart, because a sailor will evaluate the danger at 4m whereas the real danger is at 5m. A displacement of a 4m isobathymetric line towards a less deep area (e.g., a 3m area) on a chart is not allowed, because a sailor will evaluate the danger at 4m whereas the real danger is at 3m.

The remainder of the paper proceeds as follows: next section describes more precisely previous works on the detection of spatial conflicts in a large set of B-spline curves. Neighborhood information as well as closeness information will be deduced. Section 3 presents geometrical and mechanical approaches for curve displacement and discusses on their interests. In section 4, a technique adapted for cartographic generalization is proposed. A deformation strategy based on both geometrical and mechanical methods which satisfies the whole constraints (i.e., safety and legibility) is applied. Some results obtained on real cases are presented.
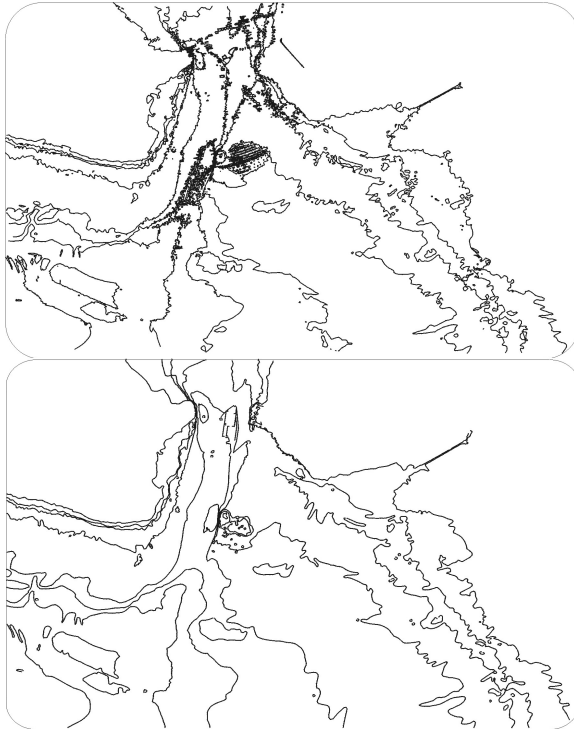

Figure 1. An example of isobathymetric lines before (above) and after generalization.

2. PREVIOUS WORKS ON THE LOCATION OF INTERSECTIONS

Let us first introduce the following notations. The B-spline curves of the charts are denoted by f^i with $0 \le i \le N$ and are of order k (in this paper, k=4). A curve f^i has $m^i + 1$ control points Q_j^i with $0 \le j \le m^i$. Its knots are denoted by $(t_j^i)_{j=0}^{j=m^i+k}$. The first and last knots have multiplicity k (clamped knot vector).

In [Gui03], a hierarchical decomposition is proposed that splits both the space and its curves in order to detect potential spatial conflicts. Then, a refinement technique to confirm or reject the intersection is applied.

The interest in our approach is twofold. Firstly, limiting the calculus to point coordinates comparison reduces the number of computations. The method can thus be applied to large set of curves. Secondly, all kinds of intersections (i.e., transversal intersections, overlapping and also visual intersections) are detected with the same process. Thus, the method can be used for complex problems such as cartographic generalization.

Hierarchical decomposition

The principle is based on two simultaneous processes.

The first process enables us to define areas where there are potential intersections according to the legibility constraint. The choice of a uniform partitioning like a quadtree depends on the two main objectives of reducing the computing time and the memory space.

The initial chart being identified as the root (i.e., initial cell) of the quadtree structure, each cell is split (or not) according to the following criterion: if the cell contains no more than one curve segment, no intersection can occur and the segmentation process stops. Otherwise, there is (a) potential intersection(s) and the cell is split. The segmentation process can also stop if:

- A cell minimal size ε_{cell} $(\varepsilon_{cell} > \varepsilon_{vis})$ is reached according to the fact that the smaller the cell, the more precise the location;

- The curve segments are defined with k control points (i.e., the order of the curves). A straightforward decomposition is no more possible. Splitting the curve corresponds to a Bézier curve subdivision and is avoided.

Figure 2 displays the result obtained by the application of the space partitioning method on Figure 1 chart.
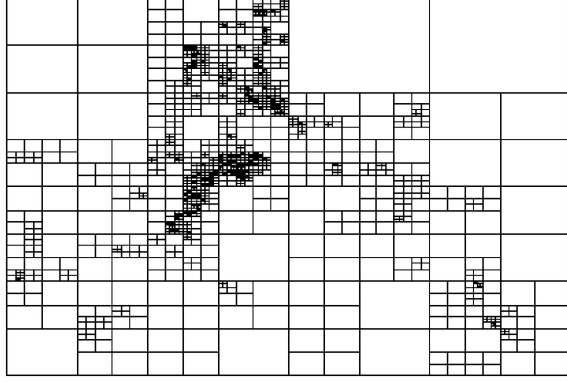
Figure 2. Segmentation of the initial chart (Figure 1) using a quadtree.

The second process splits the different curves f^i according to the spatial segmentation. The aim is to link each cell to the smallest parametric intervals $[t^i_{first}, t^i_{last}]$ (that defines the parts of the curves f^i) intersecting it. To reach this goal, a method based on a removal of parametric intervals [Dan92] is applied. The interest in this method is that no point on the curve is computed and no new knot is inserted, which reduces the computing times.

At the end of this hierarchical decomposition process, a quadtree is defined where each cell contains a list (empty or not) of B-spline curve segments.

There are potential intersections when there are at least two curve segments in the same cell. The refinement method described in the next section allows us to confirm the intersection.

Refinement technique

The aim of this section is to validate the intersections between different curves in a same cell. The approach is geometrical: the curve segments are bounded by thin envelopes which are compared.

A subdivision scheme [Lut00] is applied on each curve segment. This defines a tight linear envelope (Figure 3) with points $f(\varsigma_j)$ and $l(\varsigma_j)$, where:

- ς_j are the Greville abscissae (i.e.
  $$\varsigma_j = \frac{t_{j+1} + \ldots + t_{j+k-1}}{k-1});$$
- $l(\varsigma)$ is the piecewise linear interpolant of the control points at Greville abscissae (i.e. $l(\varsigma_j) = Q_j$).

On a parametric interval $[\varsigma_j, \varsigma_{j+1}]$, Lutterkort and Peters prove that the bandwith of this envelope is bounded by:

$$\|f - l\| \leq \frac{k}{24} \max_{j - \frac{k-1}{2} + 1 \leq i \leq j + \frac{k-1}{2} - 1} \|Q_{i-1} - 2Q_i + Q_{i+1}\|_2 \quad (1)$$
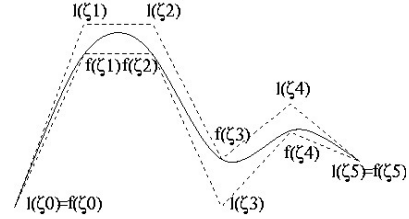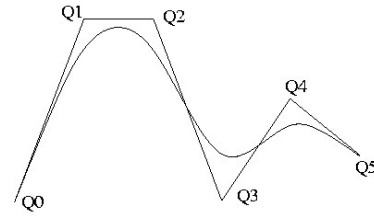

Figure 3. Tight envelope of a B-spline curve. Above: control polygon. Bottom: envelope

One can previously apply the following subdivision scheme to the initial control points to have a thinner envelope:

$$Q_{2j}^{(n+1)} = \frac{1}{2}(Q_j^{(n)} + Q_{j+1}^{(n)})$$
$$Q_{2j+1}^{(n+1)} = \frac{1}{8}Q_j^{(n)} + \frac{3}{4}Q_{j+1}^{(n)} + \frac{1}{8}Q_{j+2}^{(n)} \quad (2)$$

The number of iterations needed to have a predefined bandwidth ε_{band} is given by:

$$n = \log_4 \frac{k \max_{0 \leq j \leq m-1} \|Q_{j-1} - 2Q_j + Q_{j+1}\|_2}{24\varepsilon_{band}} \quad (3)$$

We assume that two curve segments intersect if the minimal distance between their envelopes (i.e., the minimal translational distance (MTD) of the Minkowski difference between the two envelopes) is less than the legibility distance ε_{vis}. When different intersections involving the same curves are detected on neighboring cells, one merges the corresponding parametric intervals of each segment in order to identify a single conflict.

In [Gui03], some results are presented and analyzed with different quadtree depths (i.e., ε_{vis}) and different approximation errors (i.e., ε_{band}).

We now focus in the following on the problem of correction with displacement techniques.

3. RELATED WORKS ON DISPLACEMENT ALGORITHMS

Geometric Methods

Displacement algorithms are of two kinds. The first of them are based on geometrical criteria. Most of them process with polygonal curves. Let P_i^0 and P_i, $0 \leq i \leq n$, be the sets of points before and after correction and

$$\Delta P_i = P_i - P_i^0 \qquad (4)$$

be the corresponding displacements. The aim of geometric methods is to determine accurate ΔP_i that solve the conflicts. These displacements are expressed according to the application constraints.

As an example, Nickerson [Nic88] introduced a classification of conflicts and their corresponding displacements. But the algorithm is restricted to the vicinity of a conflict and cannot manage displacements if more than two lines are in conflict.

A global method taking into account the neighboring curves is presented by Harrie [Har99]. In this paper, constraints are applied to the points defining the lines. Each constraint is expressed as a linear equation:

$$\sum_{i=0}^{n} c_i \, \Delta P_i = \text{constant} \qquad (5)$$

The weights c_i are used to set the significance of constraints. If necessary, non linear constraints are formulated via Taylor developments. For example, if a line cannot be shifted, the constraint for each point P_j of the line is:

$$\Delta P_j = 0 .$$

If this line can only be translated, we will set:

$$\Delta P_j - \Delta P_{j+1} = 0 .$$

Once all the constraints have been identified, they are expressed as a linear system with as many equations as constraints. The unknowns of the system are the ΔP_i. As we may have more equations than unknowns, the system can be over-determined so a compromising solution is found by using a least square method for minimizing the error. That means that some constraints (having the lowest weights) may not be satisfied.

The main problem with global geometrical methods as in [Har99] is to anticipate the results. The behavior of the method is not natural. Therefore, a second kind of methods has been developed based on physical models.

Energy Minimization Methods

The principle is that a model is at its equilibrium state if its energy is minimal or if the sum of all the forces applied on it vanishes. The energy of a system is shared into internal and external energies. The first part represents the inner energy of an object and tends to preserve its shape. The second part is the energy brought by other objects or constraints which penalizes the system and tends to deform it. Active contours and cable networks pertain to this category of methods. In this paper, we focus on cable networks.

3.1.1 Cable networks

A physical model is presented by Léon in [Léo95]. The principle is to consider a cable network and to deform it by changing mechanical parameters such as the tension in the cable or by applying external forces to the nodes of the network. For B-spline deformation, Léon suggests using the analogy of the standard representation of a control polygon (for curve) or control polyhedron (for surface) with a cable network. The deformation is obtained through modifications applied to control points. Such a mechanical deformation has been applied to the definition of some line generalization operators [Sau98].

Each segment of the control polyhedron is a cable under tension where nodes can be either fixed or free. The equilibrium shape is reached when all the forces at each free node sum up to zero.

Let P_i be the nodes of the network and b_i be the branches joining the points P_i and P_{i+1}. If F_i^{ext} is the load applied to a node P_i and F_i^{int} the internal force in the bar b_i, F_i^{ext} equilibrates the sum of all the forces that meet at P_i:

$$F_{ix}^{ext} = \sum_k F_{kx}^{int}$$

$$F_{iy}^{ext} = \sum_k F_{ky}^{int}$$

The internal forces F_i^{int} of each branch b_i can be replaced by a force density defined by $q_i = \dfrac{F_i^{int}}{d_i}$

with d_i the length of the branch. The equivalent matrix which is built from the force densities is written as Q:

$$Q_{ij} = \begin{cases} q_i & \text{if } i=j \\ 0 & \text{otherwise} \end{cases} .$$

The connectivity of the network is expressed by the branch-node matrix C defined by the equation

$$C_{ij} = \begin{cases} 1 & \text{if bar } i \text{ ends at point } j \\ -1 & \text{if bar } i \text{ ends at point } j \\ & \text{and other end is already 1} \\ 0 & \text{otherwise} \end{cases}$$

This matrix is divided into two others C_f and C_l by grouping the columns among fixed points and free points. Equilibrium equations can also be written as a linear system established under the following matrix form with $D_l = C_l^t Q C_l$ and $D_f = C_l^t Q C_f$:

$$\begin{cases} D_l x_l = F_x^{ext} - D_f x_f \\ D_l y_l = F_y^{ext} - D_f y_f \end{cases} \quad (6)$$

where x_f, y_f are the coordinates of the fixed points and are considered as input data while x_l, y_l are the coordinates of the moving points and the unknowns of the system. The advantages of the force density method are straightforward: the computation of the point coordinates results from the solution of a linear system and as the densities are positive, the solution found is unique and always exists.

Energy minimization methods have a better behavior than geometric methods because they are defined with apprehensive values. The interest of cable network is that it needs only the resolution of a linear system to compute node position according to the forces.

4. CORRECTION OF INTERSECTIONS

Correcting an intersection between two isobathymetric lines can be obtained by combining a geometrical and a mechanical methods. The geometrical approach yields a valid solution solving all the spatial conflicts. However, the displacement may be too important. A mechanical method is then applied to slightly modify the curve in order to get closer to the initial one.

The security constraint (see section 1) requires moving only the deepest line along a vector u indicating the direction of deeper areas (Figure 4).

In the following, we search the minimal displacements on the subdivided polygon to solve the conflicts. These displacements are applied to the original control points.

Computation of Minimal Displacements

We assume that the technique described in section 2 has located a conflict in adjacent cells between different curve segments. Determining the minimal displacement is realized by using the tight envelopes of the curve segments in conflict computed in section 2.

Let l_0 and l_1 be the two closed polygonal lines defining the tight envelopes of the two curve segments s^0 and s^1, with s^1 the curve segment to be deformed. The points of l_0 and l_1 are P_i^0 and P_j^1.
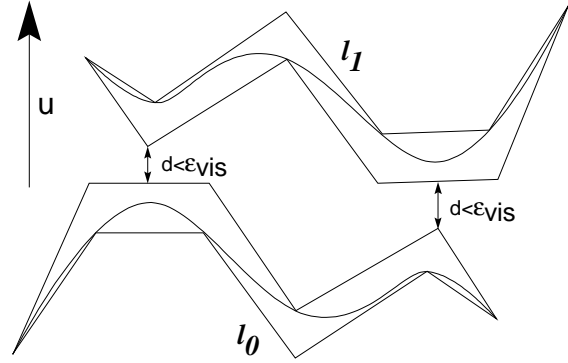

Figure 4. An example of conflicting curves.

There is a conflict between l_0 and l_1 if the distance between two segments $P_j^1 P_{j+1}^1$ and $P_i^0 P_{i+1}^0$ is smaller than the legibility distance ε_{vis}. Removing the conflict (Figure 4) can be obtained by translating $P_j^1 P_{j+1}^1$ of a vector ΔP_j^1 which corresponds to the minimal translational distance (see section 2).

Thus, to a list of conflicting segments $P_j^1 P_{j+1}^1$ corresponds a list of minimal displacements ΔP_j^1.

Two strategies can be considered. The conflict can be corrected either by shifting the points of l_1 or by shifting the initial control points Q_i of s^1. We remind the reader that the envelope l_1 is defined by points of the curve $s^1(\varsigma_j)$ and points $l(\varsigma_j)$ corresponding to control points obtained by subdividing the initial control polygon where ς_j are the Greville abscissae. For each point P_j^1, there exists ς_j such that $P_j^1 = s^1(\varsigma_j)$ or $P_j^1 = l(\varsigma_j)$: to a list of conflicting segments corresponds a list of abscissae $\varsigma_{j\min} \leq \varsigma_j \leq \varsigma_{j\max}$.

Regarding the first case, the displacements ΔP_j^1 are applied to the segments $P_j^1 P_{j+1}^1$. Once the subdivided polygon is modified, the displacement must be transmitted to the initial control polygon for preserving a homogeneous curve representation. This

stage is necessary as the curves are expressed and stored with their initial control points. That means that one has to go back up the subdivision scheme. However, this operation does not generally yield a valid solution when the points $l(\varsigma_j)$ are displaced.

Therefore, the conflict is corrected by shifting the control points of the initial curve corresponding to the parametric interval in conflict (see section 2).

The following section aims at defining a geometrical deformation that ensures the removal of all the conflicts.

Geometrical method

Let us first assume a conflict on segment $P_j^1 P_{j+1}^1$ of l_1 with $t_i \leq \zeta_j < \zeta_{j+1} < t_{i+1}$. We want to determine a ΔQ_i which corrects the conflict by shifting control points $Q_{i-k+1}, ..., Q_i$ of ΔQ_i.

The conflict is corrected if the segment is shifted of $\Delta Q_i = \Delta P_j^1$. The conflict is also corrected if the segment is shifted in another direction v where $\Delta P_j^1.v > 0$. In that case, the displacement must be:

$$\frac{\left\| \Delta P_j^1 \right\|}{\cos(\Delta P_j^1, v)}.$$

Let us now assume that we have different segments in conflict all corresponding to the same parametric interval $[t_i, t_{i+1}]$. The direction of ΔQ_i must be chosen so that the displacement along ΔQ_i is minimal and solves all the conflicts on this interval. That means that (Figure 5):

$$\left\| \Delta Q_i \right\| = \min_p \max_j \frac{\left\| \Delta P_j^1 \right\|}{\cos(\Delta P_p^1, \Delta P_j^1)}, \forall \zeta_p, \zeta_j \in [t_i; t_{i+1}] \quad (7)$$

with ΔQ_i directed along the ΔP_p^1 which minimizes equation 7.
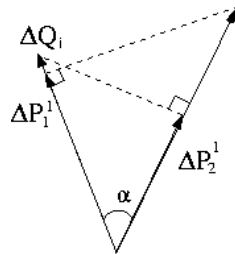
Figure 5. Computation of the minimal displacement vector on an interval: both vectors correct the conflicts but ΔQ_i is chosen as the smaller one.

We are now considering the general case. We assume that a list of conflicting segments $P_{j_{\min}}^1$, ..., $P_{j_{\max}}^1$ of l_1 has been identified. The curve segment defined by the smaller parametric interval $[t_{i_{\min}}, t_{i_{\max}}]$, where $t_{i_{\min}} \leq \zeta_{j_{\min}} < \zeta_{j_{\max}} < t_{i_{\max}}$, has to be deformed. The control points related to this interval are $Q_{i_{\min}-k+1}$, ..., $Q_{i_{\max}-1}$. A first solution is to compute, using a generalization of the process described above, a unique ΔQ to be applied to the whole control points. This approach yields unfortunately too large displacements : the larger the number of concerned control points the poorer the result. In the second solution, we compute a minimal displacement for each control point.

A control point Q_i is linked with k intervals $([t_i, t_{i+1}], ..., [t_{i+k-1}, t_{i+k}])$. k displacements to be applied on Q_i must be computed, as described above. Then, a minimal vector $\Delta \tilde{Q}_i$ is once again computed with these displacements. It ensures that all the conflicts involving Q_i are solved.

As a verification, one can remark that each displacement $\Delta \tilde{Q}_{i-k+1}, ..., \Delta \tilde{Q}_i$ takes into consideration interval $[t_i, t_{i+1}]$ ensuring a global displacement in agreement with the minimal required displacement for this interval.

An example of conflict correction is given in Figure 6. The corrected solution is called $\tilde{s}_1^{(0)}$. Its control points are points $\tilde{Q}_i^{(0)}$.
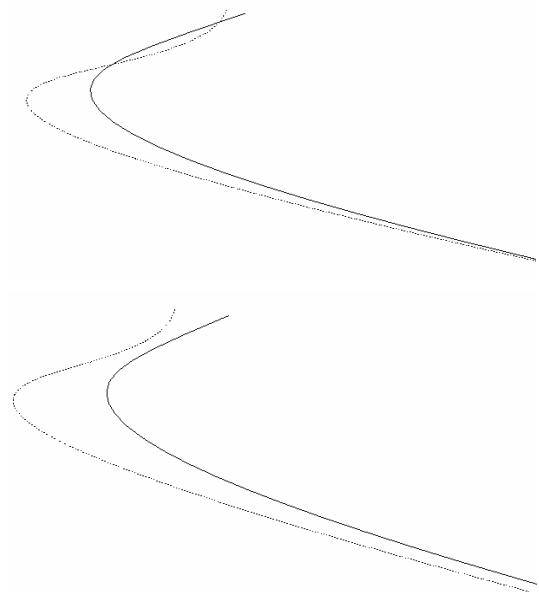
Figure 6. Above: s_0 (dashed line) and s_1 (solid line). Bottom: s_0 and $\tilde{s}_1^{(0)}$ after displacement.

Nevertheless, when angle α in Figure 5 is obtuse, the displacements can be too important. For example, in Figure 7, the conflicting segments are marked by two circles. The correction obtained with the geometrical method is shown in Figure 8. This is also due to the fact that the curve is defined with a few number of control points (16 control points). Our aim is to build a curve s_1 closer to its initial location. For that purpose, we introduce in the next section, a mechanical method.
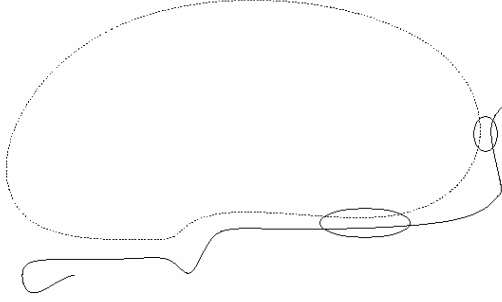

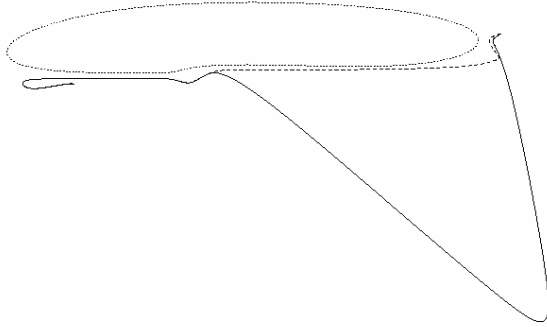**Figure 7. Conflicts between two curves**

Figure 8. An example of too large displacement (initial line: dashed line, corrected line: solid line)

Mechanical method

A mechanical method is applied in order to refine the solution. One difficulty of mechanical methods is to determine the different forces for obtaining the required displacement. The first interest of computing a geometrical solution before applying a mechanical method is that it allows us to compute the external forces deforming the curve from s_1 to $\tilde{s}_1^{(0)}$ for initializing an iterative refinement. Moreover, the control points whose positions are very close between s_1 and $\tilde{s}_1^{(0)}$ can be considered as fixed, entailing an important reduction of the complexity.

From formula 6, one can compute the external forces F_i^{ext} which bring the initial control points Q_i of s_1 to their new positions $\tilde{Q}_i^{(0)}$. The solution of the cable network is unique so, by applying forces $-F_i^{ext}$ to $\tilde{Q}_i^{(0)}$, one goes back to the original control points Q_i.

If forces $-cF_i^{ext}$ are applied with c a constant smaller than 1, a curve located between s_1 and the modified curve $\tilde{s}_1^{(0)}$ is obtained. The goal is to find the greater forces that can be applied to $\tilde{s}_1^{(0)}$ respecting the legibility constraints. In our approach, the network is homogeneous, i.e., the internal force densities are fixed. Only the external forces can be modified.

The process is iterative and is initialized with the geometrical solution $\tilde{s}_1^{(0)}$ having all its control points set free. At iteration n, forces $-cF_i^{ext(n)}$ are computed for deforming the curve $\tilde{s}_1^{(n)}$ to obtain a better solution $\tilde{s}_1^{(n+1)}$.

A criterion based on the subdivision scheme introduced in section 2 enables us to accept (no conflict with s_0) or reject the solution. By applying a dichotomy to parameter c, we search an interval $[c_{min}, c_{max}]$ such as

-     c_{min} yields a valid solution;

-     c_{max} yields a non valid solution.

The dichotomy is applied until either a given width for interval $[c_{min}, c_{max}]$ or a number of iterations is reached. The new curve $\tilde{s}_1^{(n+1)}$ is obtained by applying forces $-c_{min}F_i^{ext(n)}$ to $\tilde{s}_1^{(n)}$. The control points related to the conflicting curve segments regarding c_{max}, are set fixed in $\tilde{s}_1^{(n+1)}$. By fixing new control points, the external forces which equilibrate the network must be recalculated.

The algorithm is repeated until all the control points are fixed. At this point, the curve cannot be modified anymore. An example of mechanical modification is shown in Figure 9.
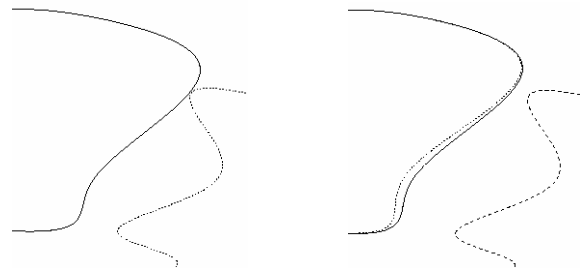

Figure 9. Correction of a conflict. Left: conflicting curves. Right: geometric (dashed line) and mechanical (solid line) solutions

The case of Figure 8 can be corrected with this method and yields to the solution given in Figure 10.
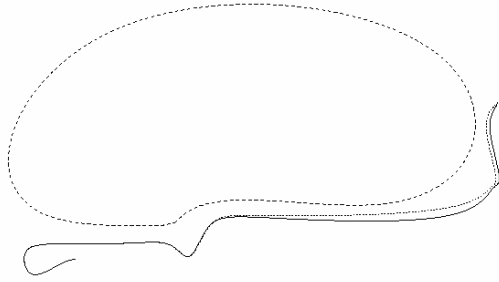
Figure 10. Mechanical solution of the curve of Figure 8 (s_0 : dashed line, s_1 : dotted line, corrected line: solid line).

5. FURTHER WORKS

A method for the correction of visual and real intersections between B-spline curves has been introduced. This method is applied to the correction of conflicts in cartographic generalization. First of all, the location algorithm has been presented. Then, different methods for the shape modification of lines are detailed. One can use either geometrical or mechanical model to calculate the displacements. Our method utilizes both of them.

Firstly, the conflict is corrected geometrically by displacing control points of the deepest curve. A valid solution is received but it may be locally too far from the initial curve. Secondly, a mechanical method is used. The method consists in applying forces on the control points in order to slightly modify the curve. The process is iterative and is repeated as long as the curve can be pushed without creating a conflict.

The method gives satisfying results on real cases but some improvements may be done. Particularly, the tuning of external forces is still an open problem and some finest criteria may be found, to preserve the shape or to minimize the displacement. The definition of new external forces directions as well as different internal forces constrained by deformation energies is the subject of our future works.

Other deformation models such as active contours must be studied. They have been used for the generalization of roads defined with polygonal lines in [Bur97, Bad01] and for the approximation of curves with B-splines in [Pot02].

Finally, we will focus on conflicts involving more than two curves. One solution can be to define a displacement area in which the curve must be shifted.

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