Old Movies Noise Reduction via Wavelets and Wiener Filter

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ABSTRACT

An effective 3D motion compensated extension of *WienerChop* model [Gha97a, Cho98a] for image sequences is presented. The proposed framework is based on a suitable combination of Wiener filter and wavelet transform and it can be shown to outperform the currently available techniques. Effectiveness combined with a low computational effort makes this approach very attractive and then it can be successfully employed in restoration of old movies.

Keywords
Image sequences, Noise reduction, Wiener filter, Wavelets, Old movies.

1   INTRODUCTION

In the last few years a great interest has been devoted by many researchers to restoration of digitized old movies which represent a cultural treasure to be preserved. Even though various events give rise to film degradation, in this paper we will deal with reduction of noise mainly due to degradation of the original film material or bad transmission — a good review can be found in [Kok98a]. Fastness and user's independence play a fundamental role in designing models for Noise Reduction of Old Movies (*NROM*) since a large number of degraded frames has to be processed (86400 for one hour of film). Standard 3D denoising problem can be represented as follows:

$$g_k(i,j) = f_k(i,j) + n_k(i,j) \qquad (1)$$

$$i = 1, .., N_1; \quad j = 1, .., N_2; \quad k = 1, .., J$$

where $\{g_k\}$ is the observed image sequence composed of J frames of size $N_1 \times N_2$, $\{f_k\}$ is the original one while $\{n_k\}$ is a zero-mean gaussian noise with variance σ^2 ([Kok98a, Boo98a, Fan99a, Bra95a]). As a matter of fact old movies are also affected by film grain noise which is signal dependent [Boo98a]. Nevertheless, using an orthogonal operator and some arrangements, it can be processed like the gaussian one.

With regard to *NROM*, in chap. 10 of [Kok98a] Kokaram proposed a motion compensated *3D FIR Wiener filter*, showing better performances than existing Wiener filtering based approaches. As a matter of fact, Wiener filter has been strongly employed in image noise reduction, thanks to its both theoretical and practical effectiveness. In fact, if on one hand it can be rigorously proved that ideal attenuation outperforms ideal selection (see [Mal98a] p. 437), on the other hand empir-

ical attenuation based models achieve very interesting results [Bru02a]. Since its original formulation [Mal98a], Wiener filter achieves better performances in the Karhunen-Loève basis. Unfortunately, its high complexity along with the non-gaussian property of real images lead to an approximation of that basis. Therefore different schools of thought appear in literature but wavelet based approaches seem to be the most followed thanks to the signal compaction properties of a wavelet representation. Recently, in [Bru02a, Bru03a] several hybrid methods using Wiener filter in a wavelet domain have been compared showing that *WienerChop* approach [Gha97a, Cho98a] represents the best combination between wavelet representation and Wiener filtering. In fact, it is at the same time effective (in terms of *Signal to Noise Ratio* (SNR) values), fast and completely automatic, requiring a very low computing time. All these properties make this approach very attractive for being used in the restoration of old movies.

The contribution of this paper consists of presenting: i) a 3D motion compensated generalization of *WienerChop* framework, ii) some theoretical results about criteria concerning the choice of the involved wavelet bases and iii) its great robustness to erroneous motion estimation.

The outline of the paper is the following. Section 2 focuses on a short description of *WienerChop* approach along with its 3D generalization. Some experimental results with some comparisons are presented in Section 3 while Section 4 draws the conclusions.

2   3D EXTENSION OF *Wiener-Chop*

As previously mentioned, the main features of an ideal *NROM* model are user's independence, effectiveness, fastness and robustness to motion estimation errors. Therefore, as claimed in the Introduction, a suitable 3D extension of *WienerChop* model can result a good proposal for reducing noise in old movies.

2.1   *WienerChop*

WienerChop is a scheme for designing an empirical Wiener filter in a wavelet domain [Gha97a, Cho98a]. Let g be a noisy image, i.e.

$$g(i,j) = f(i,j) + n(i,j),$$
$$i = 1, ..., N_1, \ j = 1, ..., N_2 \qquad (2)$$

where f is the original image and n is a zero-mean gaussian noise. Let f^h be the hard thresholded

estimate of the signal f in a wavelet basis W_1. If the wavelet transform W_2 is performed in eq.(2), for a fixed scale s we have

$$y(i,j,s) = x(i,j,s) + z(i,j,s) \qquad (3)$$

where $y = W_2 g$, $x = W_2 f$ and $z = W_2 n$. Hence, the empirical Wiener filter can be designed as

$$h_w(i,j,s) = \frac{\hat{x}^2(i,j,s)}{\hat{x}^2(i,j,s) + \sigma^2} \qquad (4)$$

where σ^2 is the noise variance while $\hat{x} = W_2 f^h$. Then the restored wavelet coefficients can be computed as follows

$$W_2 \hat{f}(i,j,s) = h_w(i,j,s) y(i,j,s) \qquad (5)$$

and the restored image is obtained simply inverting the operator W_2. The block scheme of *Wiener-Chop* is depicted in Fig. 1(left).

As cleverly suggested in [Cho98a], the error of the estimation in eq.(5) can be written as $E_{tot} = E_{opt} + E_{mis}$. E_{opt} is the *optimal* error and it is tied to the ideal Wiener filter approximation error and then to the ideal basis selection. This latter is still an open problem and then it can be only dealt with by empirical approaches [Mal98a]. On the contrary, it is possible to reduce the *mismatch* error E_{mis}. In fact, it accounts for the mismatch of the signal model f^h to the true signal f, which, in turn, is tied to the choice of the bases to be adopted. Although this choice is empirically made in [Cho98a], some theoretical criteria for this selection are given in Appendix A.

2.2   The proposed 3D Extension

The simplest and thoughtful *WienerChop* 3D extension is depicted in Fig. 1(right). Roughly speaking, an image sequence is split in subsequences: non overlapped Group of Frames (GOF) [Boo98a]. This is performed according to the motion vector (several frames for slow and regular motion sequences, few frames vice versa). Each GOF is firstly motion compensated obtaining $\overline{GOF}$. Then its frames are processed by the first wavelet basis W_1, and correspondent band coefficients are temporally averaged and spatially hard thresholded (the choice of the wavelet bases will be discussed later). Finally, the inverse wavelet transform W_1^{-1} is performed on them, in order to get a common clean frame which is enlarged to the size of the global scene. The latter represents the signal model to be used in the Wiener filtering of the second phase as sketched in eq.(4). In fact, each frame of $\overline{GOF}$ is decomposed in the second wavelet basis W_2 and attenuated by the aforementioned Wiener filter using

the common clean frame as signal model. An undecimated wavelet decomposition [Mal98a] has been adopted in both phases to ensure shift invariance and then reduce artefacts in the recovered sequence. In addition, in the spatial hard-thresholding step, Donoho's *universal* threshold T is employed (p. 438 of [Mal98a]). It is reduced by a factor N (where N is the number of frames of the considered $\overline{\text{GOF}}$) since wavelet coefficients have already been smoothed by the former temporal averaging. The choice of *universal threshold* would seem in contrast with the signal dependency of the film grain noise, i.e. it is proportional to signal intensity (p. 288 of [Boo98a]). Nevertheless in this case a simple arrangement of Wiener filter coefficients $(h_w(i, j))$ in the second phase gives satisfactory results. In fact, thanks to the orthogonality of the wavelet operator, film grain noise is proportional to the local mean of the original image that can be estimated from the wavelet approximation component.

We outline that the proposed model has a great robustness to erroneous motion estimation. To show that, without loss of generality, let us consider two frames showing a horizontally moving white square on a dark background. Without motion compensation, a simple averaging of the degraded frames yields a blurred estimate in the leftmost and rightmost regions of the square. A similar result is achieved by any linear transform combined with a mean. On the contrary, in our model this effect is attenuated using two bases. In fact, the aforementioned blur only involves the output of the first step of our model. Moreover this latter only regulates the attenuation of high frequency bands in the second filtering while leaving low frequencies unchanged. Keep in mind that the second wavelet decomposition is performed for each frame at a time, so that low frequency band contains a right information. Thus, this slighter attenuation in the "moving regions" leads to a reduced blur and then a better contrast. Hence, higher SNR (Signal to Noise Ratio) ratios are achieved as well as a better visual impact, in agreement with the *Weber's law* [Gon02a].

3 EXPERIMENTAL RESULTS

We have tested our model on many image sequences. Some results achieved in old movies restoration will be shown. Nevertheless, for this kind of sequences only subjective (visual) comparisons can be made because original sequences are unknown. Therefore, in order to perform some objective comparisons, we will also show the results achieved on two grey-level video test sequences: 400×512 "Mobcal" sequence [Kok98a] (22 frames)

and 480×720 "Calendar Train" sequence [Boo98a] (40 frames).

With regard to the first one, we have used the 22 db degraded image sequence contained in [Kok98a] which is corrupted with an additive zero-mean gaussian noise. Results are presented in terms of *ISNR* (Improved Signal to Noise Ratio), defined as follows:

$$ISNR(k) = 10\, log_{10} \left(\frac{\|\mathbf{g}_k - \mathbf{f}_k\|_2^2}{\|\hat{\mathbf{f}}_k - \mathbf{f}_k\|_2^2} \right) \ k = 1, \ldots, J, \tag{6}$$

where $\| \cdot \|_2$ is the Euclidean norm in $\Re^2$ while $\mathbf{g}_k$, $\mathbf{f}_k$ and $\hat{\mathbf{f}}_k$ are respectively the degraded, the original and the restored frame.

In all tests presented in this paper, a Daubechies wavelet basis with two vanishing moments has been used in the first phase and Haar basis for the second phase. This is in agreement with the results contained in Appendix A as well as the low complexity constraint valid for old movies restoration techniques. In fact, since the choice of the optimal wavelet basis for representing a signal is still an unsolved problem (chap. 9 of [Mal98a]), one basis is previously fixed while the other is chosen having its order close to the first one (see Appendix A). It means that some singularity points are preferred to others and then well preserved. Therefore if jump discontinuities are endowed, Haar basis has to be chosen. In addition, in both phases we use a five level decomposition, since it can be shown that Wiener filtering is asymptotically optimal for increasing scale level [Bru03a]. Noise variance is estimated by means of a median measurement at the finest scale of the wavelet decomposition of the degraded images [Mal98a]. Moreover GOFs are composed by 4 frames since it is a good compromise between slow and fast changing scenes assuring greater robustness to erroneous motion estimation of the proposed model as shown in Fig. 3. In this figure a zoom of Mobcal sequence recovered using only its background motion is depicted. Notice that the subtle circle of the toy pendulum is well recovered, even though its quite fast motion has not been considered at all.

An original frame of Mobcal sequence as well as a detail of the noisy and the recovered by the proposed model are respectively shown in Fig 4a, 4b and 4c.

We have compared our model with the *3D FIR Wiener filter* in chap. 10 of [Kok98a], since giving the best performances among *3D IIR Wiener filter*, *Temporal Wiener filter*, *Recursive Frame Averaging*, *Temporally Recursive filter* and *Frame Averaging*, as shown in Fig. 10.4 of [Kok98a]. In

order to make an objective comparison, a three level multiresolution BBM (Boyce Block Matching) motion estimation [Boy92a] has been employed, tuning the parameters as in [Kok98a] — see p. 250 of chap. 10 for details. Fig. 2 (left) shows that the proposed model on average gains .7 db on it. The same Figure depicts *ISNR* behavior achieved by the *Oriented Pyramid based Model* (*OPM*) [Roo96a]. This model results strongly dependent on the involved thresholds. In our implementation best performances have been achieved by selecting $T_{2a} = \sigma$ (σ is expressed in percentage), $T_{3a} = 3\sigma$ and $T_{4a} = \sigma/2$ using a soft thresholding scheme. Even though better results can be achieved using an additional threshold oriented to preserve spatial discontinuities, this way guarantees a slight gain in terms of db, resulting further user-dependent.

The proposed model has also been compared with the *Discrete Cosine Transform-Adaptive Wiener Filter* on the Calendar Train sequence, as in [Boo98a]. Again, the latter gives better results than *Hadamard Transform-Adaptive Wiener Filter*, *Adaptive Filter* and *Temporal Average Filter*. *ISNR* comparison is shown in Fig. 2 (right), where in this case the proposed model gains about 1.2 db on average, on a 20 db corrupted sequence with an additive zero-mean gaussian noise.

Finally with regard to real degraded sequences of old movies, signal dependent arrangements have been done: visual quality seems to be particularly satisfactory as shown in Figs 5 and 6 even when motion has not been considered at all.

From previous results, it turns out that the proposed model performs very well from both an objective and subjective point of view. It does not require any threshold tuning as *OPM* does, allowing us, at the same time, to avoid some undesirable side effects like black borders in *3D FIR Wiener filter*. Finally, an interesting advantage of the proposed model stems from its computational effort. In fact, it is possible to show that even though our model involves a wavelet decomposition, the total amount of required operations is more advantageous than the *3D FIR Wiener filter* alone. If we consider a sequence composed of J frames of size $N_1 \times N_2$, the Kokaram's model complexity is $O(JN_1N_2(\overline{N}^2 M^2 + \overline{N}\log(\overline{N}M^2)))$ bearing in mind that $\overline{N}$ is the number of frames required in the 3D Wiener filter design and $M \times M$ is the size of the blocks in which the frame is partitioned. On the contrary, the proposed model requires $O(JN_1N_2\max(S_1, S_2))$, where S_1 and S_2 are respectively the order of the first and second wavelet basis filter. It is obvious that a suitable choice of the wavelet bases, i.e. with few

vanishing moments, strongly reduces the computational effort, making the proposed model very attractive. Moreover its performances could be improved using the modified *WienerChop* presented in [Bru03a]. In fact it would better preserve edges suitably combining signal expansions in three wavelet bases but paying a higher computational effort. In particular, it would be able to reduce some blurring effects around small image components, such as some letters and numbers on the calendar of Fig. 4c. These effects are mainly due to the use of a universal threshold for all processed frames. Moreover small errors in motion estimation are unavoidably visible on such small details. Therefore a further improvement can be achieved by using a finer motion estimation algorithm.

4 CONCLUSIONS

A proposal for a 3D extension of *WienerChop* model for noise reduction in image sequences has been presented. Its better performances than currently available techniques have been shown. Some of its key issues, such as optimality constraints for the choice of the wavelet bases, low complexity and user's independence, are also discussed and make it particularly effective for old movies restoration.

APPENDIX A: SELECTING WAVELET BASES

The wavelet transform of a function $f \in C^n$ is

$$Wf(u, s) = \int \sum_{k=m}^{n} \frac{f^{(k)}(t_0)(t - t_0)^k}{k!} \psi(\frac{t - u}{s})dt,$$

where $f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$ is the Taylor expansion of f and ψ is a wavelet having $m \leq n$ vanishing moments. Let ψ_1 and ψ_2 be two wavelet functions having m_1 and m_2 vanishing moments ($m_1 \leq m_2 \leq n$) respectively, with supports Ω_1 and Ω_2 ($\Omega_1 \subseteq \Omega_2$). If $f_2(x) = \sum_{k>m_1} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$, then

$$|W_2f(u, s) - W_1f(u, s)| \leq$$

$$\leq \frac{1}{\sqrt{s}}|\int_{\Omega_2} f_2(t)(\psi_2(\frac{t - u}{s}) - \psi_1(\frac{t - u}{s}))dt|$$

and from Holder inequality it follows

$$|W_2 f(u,s) - W_1 f(u,s)| \leq \|f_2\|_2 \, \|\psi_2 - \psi_1\|_2 .$$

Hence

$$\|W_2 f - W_1 f\|_2^2 \leq \|f_2\|_2^2 \, \|\psi_2 - \psi_1\|_2^2 \, |\Omega_2|. \quad (7)$$

Considering that

$$\|\psi_2 - \psi_1\|_2^2 = \int_{\Omega_2} |\psi_2 - \psi_1|^2 =$$

$$= \int_{\Omega_2 - \Omega_1} |\psi_2|^2 + \int_{\Omega_1} |\psi_2 - \psi_1|^2 =$$

$$= \|\psi_2\|_{\Omega_2 - \Omega_1}^2 + \|\psi_2 - \psi_1\|_{\Omega_1}$$

and $|\Omega_2| = |\Omega_1| + |\Omega_2 - \Omega_1|$, (7) can be trivially rewritten as

$$\|W_2 f - W_1 f\|_2^2 \leq$$

$$\leq \|f_2\|_2^2 \, (\|\psi_2\|_{\Omega_2 - \Omega_1}^2 \, |\Omega_2 - \Omega_1| + \|\psi_2 - \psi_1\|_{\Omega_1}^2 \, |\Omega_1|)$$

$$+ \max_{\Omega_2}(|f_2|^2) \, (\, (\|\psi_2\|_{\Omega_2 - \Omega_1}^2 - \|\psi_2 - \psi_1\|_{\Omega_1}^2) \, \frac{|\Omega_1|}{|\Omega_2|} \, +$$

$$+ \|\psi_2 - \psi_1\|_{\Omega_1}^2 \,).$$

It follows that the smaller $|\Omega_2 - \Omega_1|$ and $\|\psi_2 - \psi_1\|_2$, the smaller the error of the representation, supposing that ψ_2 is the best decomposition basis for the signal f.

This result is in agreement with the following one ([Mal98a] Prop. 10.2): *for a piecewise polynomial with K breakpoints, a hard-thresholding estimator calculated with a Daubechies wavelet with $d+1$ vanishing moments, satisfies $E\{\|\hat{f} - f\|^2\} \leq \sigma^2 K(d+1) C \frac{\log^2 N}{N}$ (N is the signal length, C is a constant and σ^2 is the noise variance).* In fact $m = d+1$ is the optimum number of vanishing moments for that kind of signal. Different choices of m yield an increasing approximation error.

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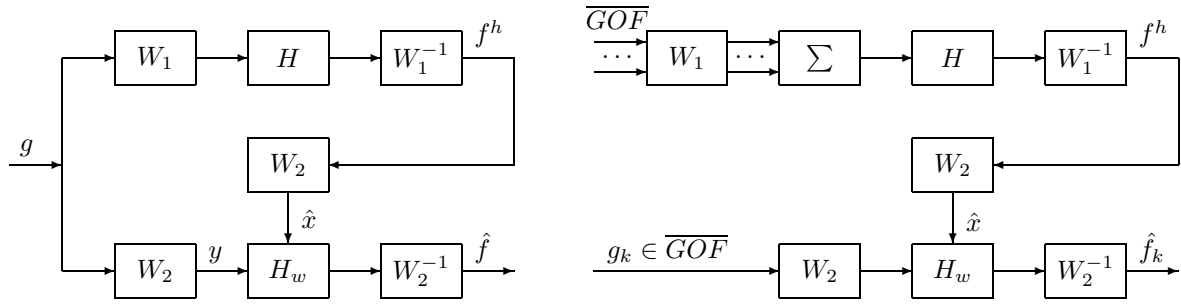
Figure 1: Block scheme of *WienerChop* model (left) and its 3D extension (right).
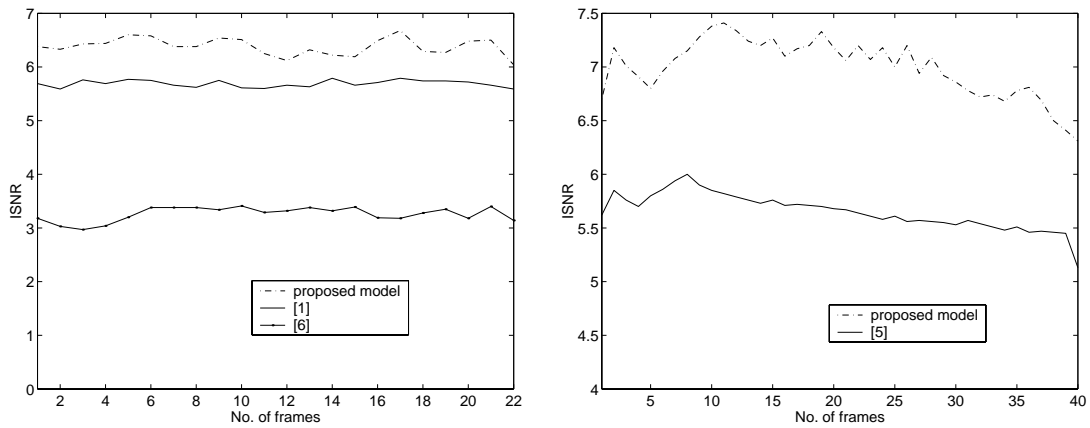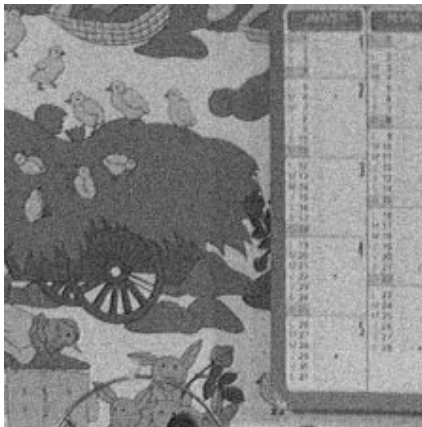

Figure 2: *ISNR* comparison: left) versus 22 frames of Mobcal sequence (SNR = 22db), right) versus 40 frames of Calendar Train sequence (SNR = 20db). Keep in mind that [1] = [kok98a], [5] = Boo98a and [6] = Roo96a.

Figure 3: Mobcal sequence: zoom of recovered toy pendulum using only background motion.

Figure 4: Mobcal sequence: a) original frame, b) noisy detail, c) recovered detail by the proposed model.

Figure 5: Knight sequence: left) 35th noisy frame; right) 35th restored frame. Notice that the image has been only denoised and then it is still affected by other kinds of degradation like scratches.
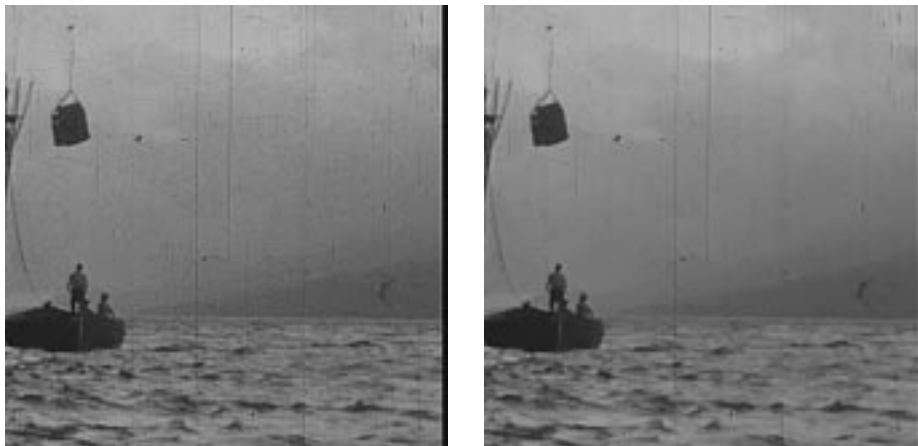

Figure 6: Sitdown sequence: left) 7th noisy frame; bottom) 7th restored frame. Notice that the image has been only denoised and then it is still affected by other kinds of degradation like scratches and blotches.