FUZZY LINKING MODELS FOR PYRAMIDAL EDGE DETECTION

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ABSTRACT

This paper presents a novel approach to multiresolution edge detection, which combines the grayscale morphological filtering, pyramid data structure and fuzzy technique. It mainly addresses the linking of edge nodes at adjacent levels in image pyramid. In previous pyramidal approaches, linking is based on linear relationship and intensity proximity only. The approach proposed here contains multiple linking mechanisms and introduces fuzzy technique. It considers the parent-child linking relationship of edge nodes between the two adjacent levels as fuzzy model, which is trained offline using real image data. Through this fuzzy linking model, the coarse, low-resolution edge map is propagated and refined to the fine, high-resolution edge map in the pyramid. The validation experiment is carried out on one synthetic image and two real images, and the results show that our approach has better performance on the localization and detection of continuous large-scale object boundaries than Canny's edge detector and other previous multiresolution approaches. In addition, the proposed approach has high computational efficiency.

Keywords: mathematical morphology, pyramid structure, fuzzy sets, edge detection, multiresolution image analysis.

1. INTRODUCTION

Recently, in the area of edge detection and image segmentation, more and more attention has focused on multiresolution, multiscale approaches because of their ability to process and analyze image at multiple levels of resolution simultaneously. A vast amount of research has been devoted to the construction of different multiresolution, multiscale representations of images, such as pyramids, wavelets and hyperstacks. The hyperstack approaches, see [De91a, Koste97a, Vinck96a], use the scales-pace of the image, and accomplish the forming of segments and boundaries by a downward projection from the root nodes, which are easy to lead to a "partial volume" result and mistakenly isolate micropatterns. Pyramids are described as special data structure, see [Biste90a, Burt81a, Meer88a, Rezae00a, Tomor95a], in which each node contains the complex information about sub-region linked to it. These approaches need iteratively to recompute the father node value and change the linking relationship between the father node and the child node. It is evident, that these kinds of strategies are time-consuming.

On the other hand, there also exists another category of algorithm, see [Chen98a, Hong82a, Jihon99a, Pacha98a], which takes the pyramid as a family of multiresolution images created from an initial image by repetitive filtering and subsampling. The coarse segmentations and boundaries are obtained on top level first. Then various methods are used to construct the subtree of links representing segmented region on the base level. But those

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algorithms, which are all based on linear linking relationship and intensity proximity only, are poor in edge localizing and not robust enough to allow segments and boundaries to be created at lower levels of the subtree. The purpose of this paper is to improve the performance of this category of pyramid method by including additional linking criteria and introducing fuzzy technique into the edge nodes linking process, and reduce time-consumed by mainly focusing on relevant regions of interest. First the proposed algorithm constructs a multiresolution pyramid structure by successive morphological filtering and subsampling of the original image. Then it considers the parent-child linking relationship of edge nodes between the two adjacent levels as fuzzy model, which is trained offline using real image data. Via the fuzzy linking model, links are created between the edge nodes of the successive levels by testing the "*similarity degree*" of edge nodes. Finally the coarse, low-resolution edge map, which is generated on the top level by conventional edge detection method, are propagated and refined to a fine, high-resolution edge map on the bottom level by this linking subtree.

This paper is organized as following: Section 2 gives a brief description of the construction of image pyramid utilizing morphological filtering. In Section 3, we give a thorough description of how to construct the nonlinear fuzzy linking model to create the linking subtree for edge nodes in morphological pyramid. The overall performance of the proposed approach is discussed in Section 4 by analyzing experiment results and comparing with other edge detection methods. Finally a conclusion of this method is given in Section 5.

2. CONSTRUCTING MORPHOLOGICAL PYRAMID

The first step in our proposed approach is to construct a pyramid data structure based on mathematical morphology. We adopt, here, flat (grayscale) morphological operators, see [Fitzp99a]. The flat erosion of a grayscale image $I(x, y)$ by a structuring element B is defined as

$$(I \Theta B)(x,y) = \min\{I(x+k, y+l) \mid (k,l) \in B\} \qquad (1)$$

Similarly, the flat dilation is defined as

$$(I \oplus B)(x,y) = \max\{I(x-k, y-l) \mid (k,l) \in B\}. \qquad (2)$$

The grayscale opening and closing are defined, respectively, as

$$(I \circ B)(x,y) = (I \Theta B) \oplus B \qquad (3)$$

and

$$(I \bullet B)(x,y) = (I \oplus B) \Theta B. \qquad (4)$$

According to this definition, grayscale openings and closings are increasing, translation invariant and idempotent, so they can be used as morphological filters. Obviously they are a kind of nonlinear filter, and can preserve the edge information, compared to other linear filters, such as Gaussian filter.

Given an image $I(x,y)$ and a structuring element B, a morphological pyramid can be defined as a collection of images, $MP = \{I_L, L = 0,1,...,N\}$, where I_L is the image at level L, downsampled from the filtered image at level L-1 using morphological filters. The choice of N determines the scales of the objects to be detected. Given that the largest texture required to be removed out of the edge map has a maximal axis width of d_1, and the smallest object required to be kept in the edge map has a minimal axis width of d_2, then

$$\log_s d_1 < N \le \log_s d_2, \qquad (5)$$

where s denotes the sampling spacing, and in general is equal to 2.

3. PYRAMID EDGE NODES LINKING

The second important step of the proposed procedure is to define the linking relationship of edge nodes between two adjacent levels. After applying conventional edge detection technique, such as Sobel operator, local adaptive gradient threshold technique (see [Qiu90a]), a coarse, low-resolution edge map can be obtained on the top level. Then we consider the parent-child linking relationship of edge nodes between two adjacent levels as a fuzzy model, which takes multiple linking criteria as input and is trained offline. Through this fuzzy linking model, the coarse, low-resolution edge map is propagated and refined to the fine, high-resolution edge map in the morphological pyramid.

Without loss of generality, we define sampling spacing as $s = 2$. For each parent edge node in level L, we only search a corresponding 3×3 subregion of candidate child edge nodes on level L-1 for the child edge nodes that have more *similarity degree* evaluated by a fuzzy linking model. We construct this fuzzy linking model as following.

3.1. Defining Fuzzy Input and Output Variables

The first fuzzy input variable x_1, *logical distance* from object boundary, is defined by

$$x_1(i,j)=\begin{cases}0.5/f(i-i',j-j') & \text{if } i=2i' \,\&\, j=2j' \\ d_{cp}/f(i-i',j-j') & \text{if } i\neq 2i' \,|\, j\neq 2j'\end{cases} \quad (6)$$

where $d_{cp}=\max(|i-2i'|,|j-2j'|)$, (i',j') and (i,j) denote the position of the parent node and that of the corresponding candidate child node, respectively, and $i\in\{2i'-1,2i',2i'+1\}$, $j\in\{2j'-1,2j',2j'+1\}$, and $f(a, b)$ denotes the direction filtering on the node (a, b) in the binary parent edge map, where '1' denotes the edge node and '0' non-edge node. For calculating $f(a, b)$, We first define four direction masks as

$$\begin{bmatrix}0&0&0\\1&1&1\\0&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&1&0\\0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix},$$

then, if $a\neq i'$ or $b\neq j'$, $f(a, b)$ is the convolution of the binary parent edge map on the node (a, b) with the mask defined by direction of node (a, b) and (i',j'). Otherwise $f(a, b)$ equals to the sum of convolution on all four directions minus 3. Obviously, $x_1(i,j)$ is related to the distance between the node (i, j) and the center $(2i',2j')$ of the candidate child nodes and the number of edge nodes on this direction.

The second fuzzy input variable x_2, *intensity proximity*, is defined by

$$x_2(i,j)=\frac{|I_p-I_c|}{\Delta I_{\max}}, \quad (7)$$

where I_p and I_c denote the intensity of the parent and the child, respectively, and $\Delta I_{\max}$ denotes the maximum intensity difference between the parent and all their candidate children.

The third fuzzy input variable x_3, *local gradient ratio*, is defined by

$$x_3(i,j)=\frac{G(i,j)}{G_{\max Loc}}, \quad (8)$$

where $G(i, j)$ denotes the gradient of the candidate child and G_{maxLoc} the local maximum gradient of all the candidate children.

The last fuzzy input variable x_4, *global gradient ratio*, is defined by

$$x_4(i,j)=\frac{G(i,j)}{G_{\max Glob}}, \quad (9)$$

where $G_{maxGlob}$ the global maximum gradient in the image on the child level L-1.

We define one fuzzy output variable as $y\in[0,1]$, denoting *similarity degree* between the candidate child and parent. When y>0.5, this candidate child is a valid child of the parent edge node and is an edge node. Otherwise it's not.

3.2. Preparing Training Data

Preparing training data is an important step in constructing fuzzy edge linking model. First the example image pyramids are obtained by applying the method described in Section 2. And then we can extract the edge map of images on every level of pyramids based on manual outlining or semi-automatic intensity-based technique. In order to define the child-parent linking relationship between the edge nodes on the adjacent level in those example image pyramids, we introduce a fuzzy linking reference model, which has the same input and output variable as the mentioned above but only has few fuzzy sets and few fuzzy rules. When searching on the child level for every parent edge node applying the fuzzy linking reference model, we only link the child edge node to the most similar parent, where the fuzzy linking reference model has largest output, and construct a pair of input-output example data (the output is '1') for this linking relationship. Obviously, the linking relationship between child edge node and parent edge node is not exclusive, and we cannot confirm the relationship of this child edge node with other adjacent parent edge nodes, so we just don't construct the input-output data pairs for them. But for every searching non-edge node in child level, we also construct a pair of input-output example data (the output is '0').

3.3. Training Fuzzy Linking Model

After getting training data, we now train the fuzzy linking model by the following steps.

Step 1. Defining fuzzy sets for each fuzzy variable

For the above fuzzy input variables x_i in the domain interval [0, 1], we define N_i fuzzy sets, which are denoted by A_i^l, i=1, 2, 3, 4, l=1, 2, …, N_i. The shapes of the membership functions can be chosen freely and can be the same or different for each fuzzy variable. Without loss of generality, we define membership functions with triangular shapes, which cover the whole input space. The fuzzy sets on each variable are complete, that is, for each point in the range we can get at least one fuzzy set on which the value of membership function is not equal to zero. Similarly, for the fuzzy output variable y we define N_y fuzzy sets B^l, l=1, 2, …, N_y, which are complete in [0, 1], with trapezia shape membership functions.

Step 2. Constructing fuzzy rules based on Table Lookup Method, see [Wang97a].

For each four-input-single-output data pair $(x_1^k, x_2^k, x_3^k, x_4^k, y^k)$, k=1, 2, …, N, the extracted fuzzy rule can be expressed as following:

$$R^k : IF\ x_1^k\ is\ A_1^l\ and\ x_2^k\ is\ A_2^m\ and\ x_3^k\ is\ A_3^n\ and\ x_4^k\ is\ A_4^q,$$
$$THEN\ y^k\ is\ B^j$$

where A_1^l, A_2^m, A_3^n and A_4^q denote the fuzzy sets defined on the input variables x_1, x_2, x_3, x_4, respectively, and B^j denotes the fuzzy sets defined on the output variable y. They have the largest membership function output for this input-output data pair on each variable. After removing the conflicting rules, we can obtain a rule base.

Step 3. Constructing fuzzy linking model

Based on the extracted rule base, we use singleton fuzzifier, product inference engine, and center average defuzzifier, and the constructed system can be expressed as follows:

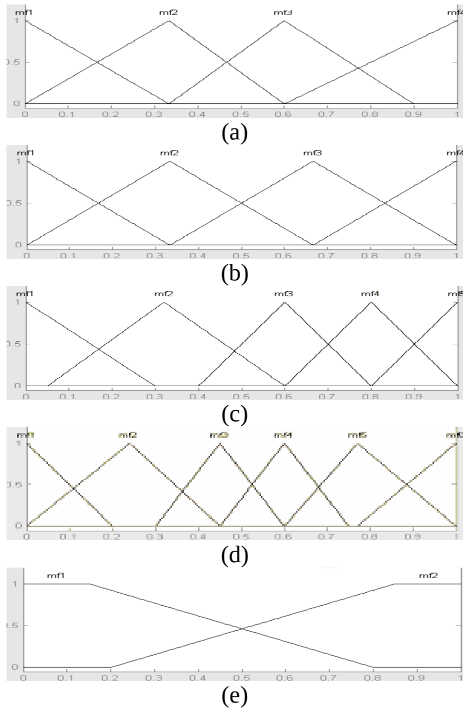
$$y = \frac{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)} \qquad (10)$$

where n is the number of input, M is the number of fuzzy rules in the rule base, and $\bar{y}^l$ is the centroid of the fuzzy set B^j, $\mu_{A_i^l}(x_i)$ is the membership function value of x_i on fuzzy set A_i^l.

4. EXPERIMENTS

First we define a simple fuzzy linking reference model, which has three fuzzy sets with Gaussian membership function for each input fuzzy variable and two fuzzy sets with trapezia shape membership function for the output fuzzy variable, and simultaneously we define 26 fuzzy rules for it. Then according to the method described in Section 3, we extract the edge maps of 10 optical images as training data. For the new fuzzy linking model, we define 4 fuzzy sets for x_1, 4 fuzzy sets for x_2, 5 fuzzy sets for x_3, 6 fuzzy sets for x_4 and 2 fuzzy sets for output variable y, whose membership functions are also defined, see Fig.1. After being trained, a new fuzzy linking model with 359 merged rules is constructed according to Eq.10.

In the following section we will give some examples applying the proposed method and the trained linking model. Without loss of generality, the sampling space used here is s = 2, and the structuring element B is a 3×3 rhombus shape window, and we use the opening operator as the morphological filtering. In order to obtain a thinner boundary, we



The membership functions for all variables. (a, b, c, d, e) The membership functions for x_1, x_2, x_3, x_4 and y, respectively

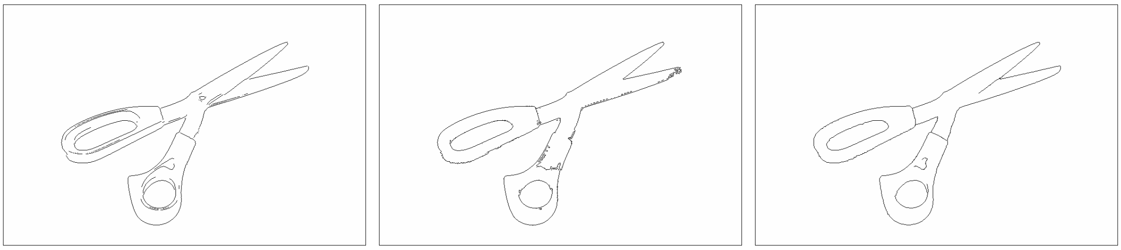
Figure 1

only take the node that has the largest *similarity degree y* in the horizontal or vertical 1×3 adjacent edge nodes as the final output edge node. In addition, the experiment results using conventional Canny's edge detector and another multiresolution edge detection method, the morphological pyramid (MP) based edge detector (see [Chen98a]), are also presented here for the sake of comparison.

Fig.2 shows two original images for the experiments in this paper. Fig.3 shows the experiment results of the proposed edge detection procedure and other two approaches applied to the *scissors* image (see Fig.2a). The edge map in Fig.3a is generated using conventional Canny's edge detector with the standard deviation $\sigma = 2$. The result using the MP-based edge detector is presented in Fig.3b, where the number of MP levels N is 8. The edge map in Fig.3c is the result of using the proposed method, in which the number of MP levels N is 4 and a Sobel detector is applied on the top level image to get the initial coarse edge map. Because the content in the original image is very simple and the contrast between object and background is very clear, so the edge maps of all three methods is good. But there is still significant difference, which the proposed method has yielded less interior texture edge and more clear exterior boundary than Canny's method, and has obtained more high edge localization

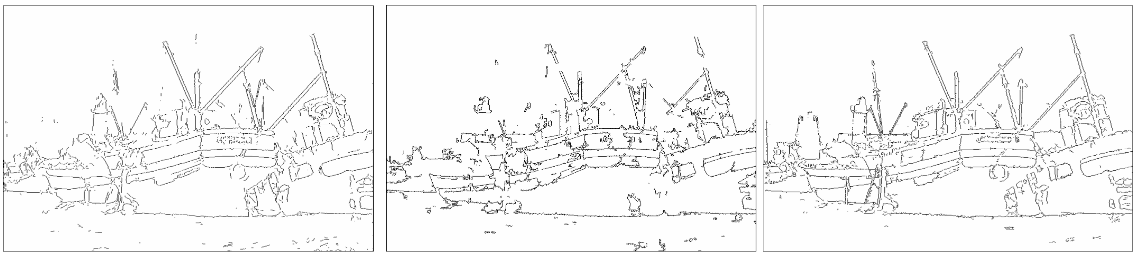
(a) (b)

Two original images for experiment
Figure 2



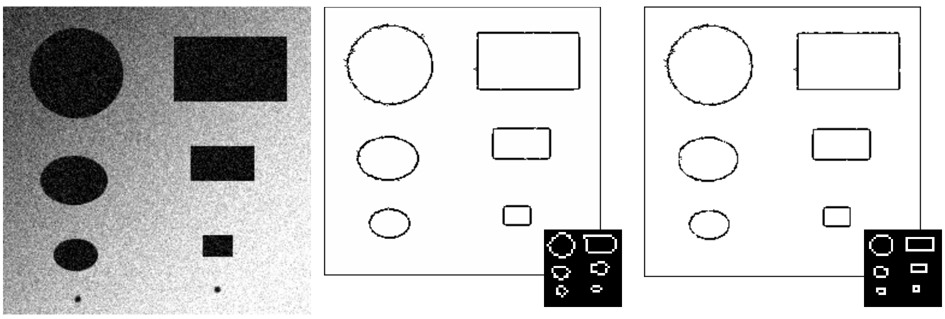
(a) (b) (c)

Experiment results of Fig.2a with the proposed method and other two methods. (a) Canny's edge detector at $\sigma = 2$. (b) MP-based edge detector. (c) The proposed method
Figure 3



(a) (b) (c)

Experiment results of Fig.2b with the proposed method and other two methods. (a) Canny's edge detector at $\sigma = 3$. (b) MP-based edge detector. (c) The proposed method.
Figure 4



(a) (b) (c)

A Gaussian noise contaminated synthetic image to verify the effect of the proposed method with different edge detection methods applied on top level of MP. (a) The original image. (b) The initial edge map on top level with Sobel edge detector and the final edge map on bottom level. (c) The initial edge map on top level with thresholding method and the final edge map on bottom level.
Figure 5

accuracy and more smooth edge map than the MP-based edge detector

The three methods of detecting the object edge are also compared for the *boat* image, see Fig.4. The edge map generated by Canny's edge detector with the standard deviation $\sigma = 3$ is showed in Fig.4a. The result of applying the MP-based edge detector is given in Fig.4b, where the number of MP levels N is 5. The edge map generated by the proposed method is presented in Fig.4c, where the number of MP levels N is 4, and we still use Sobel edge detector to extract the low-resolution edge map in top level. Obviously the proposed method can detect out more consistent boundary of large-scale object and generate less false edge than any other two methods, in such complex image.

Fig.5 demonstrates other significant features of the proposed method: strong ability to refine the boundaries and good localization accuracy. Fig.5a is a synthetic image with a gradual-changing gray background, and a Gaussian white noise with zero mean and 0.01 variance has been added to it. Using the proposed method, first a morphological pyramid of 4 levels is created on that image. Then through our fuzzy linking model, two very similar edge maps are formed on the bottom levels from the different initial coarse edge maps used in the top levels, see Fig.5b and Fig.5c, respectively. In Fig.5b the Sobel edge detector is used to extract the top edge map, while in Fig.5c first a simple thresholding method is used on the top level to get segmented image and then a binary morphological operation is used to get the top edge map. Obviously the object contours in the top levels in Fig.5b and Fig.5c are a little different (Note that they are inverted with black ground for better comparison and the boundaries in Fig.5b is not as regular as those in Fig.5c). However, through our fuzzy edge linking technique, the two edge maps on the bottom level are closely similar except a few different edge points, and very loyal to the original image. So it is easy to see that our fuzzy linking model has a very strong ability to refine the boundaries on every level and can finally obtain good boundary localization accuracy. In addition, this example has also demonstrated the proposed method has good performance in the presence of noise.

5. CONCLUSION

In this paper we have described a multiresolution approach for edge detection, which combines the morphological filtering, pyramid data structure and fuzzy technique. In our approach, first an image pyramid is constructed through repetitive morphological filtering and subsampling of original image, and then a coarse, low-resolution edge map is generated on the top level by a conventional edge detection technique. We model the parent-child linking relationship of edge nodes between the two adjacent levels using fuzzy technique and train this fuzzy model offline. Through this fuzzy linking model, the edge map is formed on the bottom level from the coarse-to-fine edge detection.

Our multiresolution edge detection approach has a number of advantages besides that it has the outstanding performance on detecting the boundary of large-scale object. Firstly, the flat (grayscale) morphological operators used in our approach are closely related to the nonlinear filtering based on statistics, which not only can preserve the edge information but also has good performance on removing noise. So it is easier to detect the coarse edge on the top level than using other linear filter in constructing image pyramid. Secondly, by fuzzy linking model, a coarse, inaccurate edge map can be refined at every level towards the real boundary of object and finally the accurate edge localization can be obtained at the bottom level just like the example in Fig.5. So it largely decreases the quality demand of edge detection method on top level. The third advantage of our approach is its computational efficiency. As we know from Eq.5, the minimum number of levels N needed to construct MP in our proposed approach only depends on the fact that the required maximum texture in the image can be removed in the edge map. So it is distinctly less than the number of levels N used in MP-based edge detector, see [Chen98a], which is related to the number of segments in the original image. Moreover, after getting the linking model having been trained offline, the parent-child linking relationship is only evaluated once in edge adjacent area in our approach, so its computation efficiency is obviously much higher than that of previous MP-based edge detector and almost the same as that of canny's method, and it can be used in real-time applications. In addition, our approach also demonstrates good performance in the present of noise.

We have used one synthetic image and two real-world optical images to validate our approach, in which only the *scissors* is ever used as training image. By visual inspection of the edge detection results, it is easy to find that our approach has better performance than the conventional Canny's edge detector and MP-based edge detector. Causing by various factors, the boundary delineated manually is not always the same with that estimated automatically in real-world image. This phenomenon is also reported by Rezaee, et al, see [Rezae00a], and this is why here we have not introduced a valid quantitative evaluation to the proposed approach, which is one of the subjects of our future studies. Also due to this reason, when constructing the training data set, we have to combine manual or semi-automatic detection method and reference fuzzy linking model to guarantee the validation of training data set. In addition, although our approach can make the coarse, inaccurate boundary finally shrinks to a stable, reliable boundary, the continuous broken points on the coarse boundary has much effect on the final edge map. It can be improved by using those edge detection methods that can get continuous boundary in the top level, such as extracting the edge from segmented image, or constructing another valid fuzzy linking model suitable for expanding searching diameter in the candidate children edge nodes. The latter also is another subject of our future studies.

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