

Theta Function Framework for Quantum Systems with Magnetic Flux

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ABSTRACT

We develop an analytic framework for quantum systems defined on a circle threaded by magnetic flux, generalizing the Bargmann representation via Theta functions. By introducing flux-dependent coherent states and displacement operators, we construct entire analytic wavefunctions and derive a reproducing kernel with quasi-periodic boundary conditions. This approach captures topological effects such as the Aharonov–Bohm phase and provides tools relevant for mesoscopic rings and topological quantum computation.

Compared to flux-free analytic approaches, the present framework directly encodes quasi-periodic boundary conditions, making it suitable for simulating observables such as persistent currents and phase shifts. Future work includes numerical implementations of the Theta function framework for flux qubits, extensions to multi-particle systems, and exploration of connections to non-Abelian topological phases.

1 INTRODUCTION

Quantum systems on compact topologies, such as circles, offer deep insights into both theoretical and applied physics. Threading such systems with magnetic flux introduces topological changes in boundary conditions, leading to observable quantum effects central to mesoscopic devices, superconducting circuitry, and emerging topological quantum computing paradigms.

A foundational quantum phenomenon illustrating this is the Aharonov–Bohm (AB) effect: a charged particle encircling a magnetic flux acquires a measurable phase shift, even in regions devoid of magnetic fields, underscoring the fundamental significance of topology and potentials in quantum mechanics [15]. In recent years, research has extended these insights to mesoscopic thermoelectric devices, where AB-induced coherence enhances quantum interference in quantum-dot heat engines [16].

Persistent currents in mesoscopic rings remain a topic of active interest. Ganguly and Maiti (2025) reported that synthetic fluxes in non-Hermitian Hatano–Nelson rings, especially in the presence of correlated disorder, can amplify both real and imaginary persistent currents [17]. Similarly, Sarkar et al. (2025) demonstrated enhancements in persistent currents under quasiperiodic

non-Hermitian disorder, even with weak interactions [18]. These studies highlight the intricate interplay between topology, disorder, and flux in modern systems.

Moreover, non-Hermitian delocalization phenomena have recently been revealed where an imaginary velocity component can drive delocalization and entanglement under periodic boundary conditions [19]. Such advances underscore the growing importance of analytic tools capable of capturing flux-induced behavior in non-Hermitian contexts.

In parallel, analytic representations - such as the Bargmann formalism provide powerful perspectives by mapping quantum states into entire analytic functions. Jacobi Theta functions naturally extend these frameworks to compact domains, encoding periodic and quasi-periodic boundary behaviors. Foundational mathematical work on Theta functions and modular forms remains crucial [8, 9], as does more recent adaptation to quantum systems on $\mathbb{Z}(d)$ and circles [10, 11, 12].

However, these prior analytic methods typically assumed strict periodicity ($q(x + 2\pi) = q(x)$) and thus failed to incorporate flux-driven quasi-periodicity or flux-dependent observables. Our work bridges this gap by embedding the AB phase into the analytic representation itself. We construct analytic wavefunctions $Q_\phi(z)$ satisfying $q(x + 2\pi) = e^{i\phi} q(x)$, thus integrating flux topology at the representational level. This extension enables analytic computation of physical observables and lays the groundwork for future exploration of interacting systems, higher-genus topologies, non-Hermitian systems, and non-Abelian topological phases [13, 14].

Contributions

- Development of a flux-dependent analytic framework using Jacobi Theta functions.
- Derivation of reproducing kernels and analysis of flux-driven zero dynamics.
- Analytical computation of persistent currents and interference phases.
- Comparison with flux-free analytic methods, highlighting clear advantages.

- Outlook toward numerical implementations, many-body interactions, and topological computation models.

The paper is structured as follows. Section 2 presents the flux-dependent analytic representation. Section 3 constructs the reproducing kernel and explores zero dynamics. Section 4 details applications including persistent currents and flux qubits. Section 5 extends the discussion to non-Hermitian and topological generalizations. Finally, Section 6 concludes with a summary and future research directions.

1.1 Boundary Conditions with Flux

Consider a quantum system described by a wavefunction $q(x)$ defined on the circle of unit radius, with $x \in [0, 2\pi]$. The Hilbert space is $L^2([0, 2\pi])$ with the usual inner product.

In the absence of magnetic flux, states satisfy strict periodicity:

$$q(x + 2\pi) = q(x).$$

This ensures that wavefunctions are single-valued on the circle.

When a magnetic flux Φ threads the circle, the system acquires a topological phase via the Aharonov-Bohm effect. The boundary condition becomes quasi-periodic:

$$q(x + 2\pi) = e^{i\phi} q(x), \quad \phi = \frac{e\Phi}{h}. \quad (1)$$

The parameter ϕ represents the flux-induced phase shift. Equation (1) encodes the fundamental modification of topology by flux.

1.2 Definition of the Analytic Representation

We now construct an analytic representation of states satisfying the quasi-periodic condition. Let Θ_3 denote the third Jacobi theta function:

$$\Theta_3(u, \tau) = \sum_{n \in \mathbb{Z}} e^{i\pi n^2 \tau} e^{2inu}, \quad \Im(\tau) > 0.$$

Theta functions provide kernels that encode periodic and quasi-periodic structures.

We define the flux-dependent analytic representation $Q_\phi(z)$ of a state $q(x)$ as

$$Q_\phi(z) = \int_0^{2\pi} q(x) \Theta_3\left(\frac{x-z}{2}, i\kappa\right) e^{-i\phi x/(2\pi)} dx, \quad (2)$$

where $\kappa > 0$ is a parameter controlling the scale of the analytic domain.

1.3 Properties of the Representation

Entire Analyticity.

Since $\Theta_3(u, i\kappa)$ is an entire function of u , and the integral in (2) is taken over a compact domain, it follows that $Q_\phi(z)$ is entire in z . Thus the analytic representation maps each physical state $q(x)$ to an entire analytic function on $\mathbb{C}$.

Quasi-Periodicity.

We now prove that $Q_\phi(z)$ satisfies a flux-induced quasi-periodicity condition. Consider:

$$Q_\phi(z + 2\pi) = \int_0^{2\pi} q(x) \Theta_3\left(\frac{x-(z+2\pi)}{2}, i\kappa\right) e^{-i\phi x/(2\pi)} dx.$$

Using the periodicity $\Theta_3(u + \pi, \tau) = \Theta_3(u, \tau)$, we have

$$\Theta_3\left(\frac{x-(z+2\pi)}{2}, i\kappa\right) = \Theta_3\left(\frac{x-z}{2}, i\kappa\right).$$

Thus

$$Q_\phi(z + 2\pi) = e^{-i\phi} \int_0^{2\pi} q(x) \Theta_3\left(\frac{x-z}{2}, i\kappa\right) e^{-i\phi x/(2\pi)} dx,$$

which implies

$$Q_\phi(z + 2\pi) = e^{-i\phi} Q_\phi(z). \quad (3)$$

Equation (3) shows that the analytic representation faithfully encodes the quasi-periodicity induced by flux.

Norm Preservation.

The map $q(x) \mapsto Q_\phi(z)$ preserves Hilbert space structure through an isometry involving the reproducing kernel (derived in Section 3). Thus the analytic representation is not only mathematically consistent but also physically faithful.

1.4 Consistency with the Flux-Free Case

An essential property of any flux-dependent generalization is its reduction to the flux-free case when $\Phi = 0$.

Setting $\phi = 0$ in (2), we obtain:

$$Q_0(z) = \int_0^{2\pi} q(x) \Theta_3\left(\frac{x-z}{2}, i\kappa\right) dx,$$

which is precisely the analytic representation used in earlier work on compact quantum systems without flux [11, 12].

Thus the present framework smoothly interpolates between the flux-free analytic representations of the literature and the flux-dependent case considered here.

1.5 Physical Interpretation

The representation $Q_\phi(z)$ encodes the entire wavefunction $q(x)$ as an analytic function with quasi-periodicity (3). Physically, this means that the AB phase is built directly into the analytic structure, rather than added later as a correction.

This is crucial for applications. For example:

- In mesoscopic rings, $Q_\phi(z)$ naturally encodes the shift of energy levels and persistent currents due to flux.
- In flux qubits, quasi-periodicity enables the analytic representation to capture flux-dependent tunneling amplitudes and interference.
- In more advanced scenarios, such as higher-genus surfaces, the method generalizes to analytic structures incorporating multiple fluxes.

By embedding flux effects at the level of representation, the framework establishes a foundation for analytic calculations of topological quantum phenomena.

2 REPRODUCING KERNEL AND ZERO DYNAMICS

The analytic representation introduced in Section 2 not only provides a mapping from Hilbert space states to entire functions, but also admits a natural reproducing kernel structure. This reproducing property ensures that the analytic representation is complete and that inner products in Hilbert space can be expressed as integrals in the analytic domain. Furthermore, the presence of flux modifies the geometry of zeros of analytic functions, an aspect that reveals deep connections to topological dynamics and Floquet theory.

2.1 Reproducing Kernel Construction

Let $Q_\phi(z)$ denote the flux-dependent analytic representation defined in (2). To establish a reproducing property, we construct the kernel

$$K_\phi(z, w^*) = \int_0^{2\pi} \Theta_3\left(\frac{x-z}{2}, i\kappa\right) \Theta_3\left(\frac{x-w^*}{2}, i\kappa\right) \times e^{-i\phi x/\pi} dx. \quad (4)$$

This kernel depends explicitly on ϕ , reflecting the role of flux. It reduces to the flux-free kernel when $\phi = 0$, in agreement with previous studies [11, 12].

2.2 Reproducing Property

We now prove that the kernel (4) reproduces analytic functions $Q_\phi(z)$. Consider

$$Q_\phi(z) = \int_A K_\phi(z, w^*) Q_\phi(w) dm(w),$$

where $dm(w)$ is the appropriate measure on the analytic domain A .

The proof follows from Fubini's theorem: inserting the definition of $Q_\phi(w)$ into the right-hand side and exchanging the order of integration recovers the original representation (2). Thus K_ϕ indeed acts as a reproducing kernel.

Inner Products.

The Hilbert space inner product can be expressed in the analytic domain as

$$\langle q_1 | q_2 \rangle = \int_A Q_\phi^{(1)}(z) Q_\phi^{(2)}(z)^* dm(z).$$

This shows that the analytic representation is not only faithful but also norm-preserving, establishing it as a genuine isometry between Hilbert space and the space of analytic functions.

2.3 Translation in the Analytic Representation

A key property of the flux-dependent analytic representation is its transformation under translations in the analytic variable z . In particular, we study how $Q_\phi(z)$ behaves when the argument is shifted by a real parameter a .

Starting from the definition (2), one finds that

$$Q_\phi(z-a) = e^{iKa/2} e^{-iKz} Q_\phi(z), \quad (5)$$

where K is related to the Fourier mode of the state.

Equation (5) shows that translations in z are not purely geometric shifts but are accompanied by flux-dependent phase factors. These exponential terms encode the quasi-periodicity introduced by the magnetic flux.

Physical Interpretation.

In the physical system, translation of z corresponds to shifting the angular coordinate of the wavefunction on the circle. The additional phases represent the Aharonov-Bohm contribution from the enclosed flux. Thus, the translation property reflects how flux modifies the analytic structure and distinguishes the quasi-periodic case from the strictly periodic, flux-free setting.

3 APPLICATIONS AND COMPARISONS

The analytic representation introduced in Section 2 and the reproducing kernel developed in Section 3 provide a rich mathematical framework. In this section, we show

how these tools can be used to compute physically relevant observables in mesoscopic physics and quantum information. We focus on three main applications: persistent currents, interference phase shifts, and comparisons with flux-free analytic methods.

3.1 Persistent Currents in Mesoscopic Rings

One of the most celebrated consequences of flux in quantum systems is the emergence of persistent currents. Consider a particle on a ring of circumference 2π , threaded by a magnetic flux Φ . The Hamiltonian is

$$H = \frac{1}{2mR^2} \left(-i\hbar \frac{d}{dx} - \frac{e\Phi}{2\pi} \right)^2,$$

where R is the radius of the ring (here set to 1). The eigenenergies are

$$E_n(\Phi) = \frac{\hbar^2}{2m} \left(n - \frac{\phi}{2\pi} \right)^2, \quad n \in \mathbb{Z},$$

with $\phi = e\Phi/\hbar$.

The persistent current is obtained as

$$I(\Phi) = -\frac{\partial E_n(\Phi)}{\partial \Phi} = -\frac{e\hbar}{m} \left(n - \frac{\phi}{2\pi} \right) \frac{1}{2\pi}. \quad (6)$$

Analytic Representation.

Within our framework, the wavefunction associated with the n -th state is encoded by

$$Q_\phi^{(n)}(z) = \int_0^{2\pi} e^{inx} \Theta_3 \left(\frac{x-z}{2}, i\kappa \right) e^{-i\phi x/(2\pi)} dx.$$

The flux-dependence is explicit in the exponential factor. Differentiating the analytic phase factor with respect to Φ reproduces the persistent current formula (6), confirming the physical validity of the analytic framework.

Physical Interpretation.

Equation (6) demonstrates that persistent currents oscillate with Φ , vanishing when $\phi = 2\pi n$. This periodicity is a direct manifestation of the quasi-periodicity of $Q_\phi(z)$. The analytic representation therefore not only encodes the correct boundary conditions but also provides a calculational tool for observables.

3.2 Phase Shifts in Interference Devices

Another key application is to interference devices such as SQUIDs and flux qubits. In such systems, the AB phase manifests as measurable shifts in interference patterns.

The quasi-periodic property

$$Q_\phi(z + 2\pi) = e^{-i\phi} Q_\phi(z)$$

implies that a translation in the analytic domain introduces a global phase shift. In an interference experiment, the relative phase between two paths enclosing different fluxes becomes

$$\Delta\phi = \frac{e}{\hbar} (\Phi_1 - \Phi_2).$$

This reproduces the experimental signature of the AB effect: oscillations in current or voltage as a function of applied flux.

Application to Flux Qubits.

In flux qubits, logical states are associated with clockwise and counter-clockwise current loops. The analytic representation captures the phase difference between these states as a flux-dependent quasi-periodicity. This provides an alternative analytic perspective on flux qubit dynamics.

3.3 Comparison with Flux-Free Analytic Methods

Flux-free analytic representations, such as those studied in [11, 12], assume strict periodicity of the form $q(x + 2\pi) = q(x)$. While these approaches successfully describe systems without flux, they fail to account for AB phase shifts and flux-controlled observables.

The present framework generalizes these methods by including the exponential factor $e^{-i\phi x/(2\pi)}$ in the representation. The advantages can be summarized as follows:

- **Boundary conditions:** Flux-free methods impose periodicity, while our framework encodes quasi-periodicity induced by flux.
- **Observables:** Persistent currents and AB phase shifts cannot be computed in flux-free analytic methods, but emerge naturally here.
- **Topological effects:** Flux introduces winding numbers and modified zero dynamics, extending beyond flux-free results.
- **Generality:** The flux-dependent framework reduces smoothly to the flux-free case when $\Phi = 0$.

3.4 Comparative Summary

Table 1 summarizes the differences between flux-free and flux-dependent analytic frameworks.

	Flux-Free	Flux-Dependent
Boundary condition	$q(x + 2\pi) = q(x)$	$q(x + 2\pi) = e^{i\phi} q(x)$
Phase shifts	Absent	Encoded by $e^{-i\phi}$
Persistent currents	Not captured	Computable via $Q_\phi(z)$
Translation / dynamics	Closed orbits	Flux-modified quasi-periodicity
Applications	Compact systems	Mesoscopic rings, flux qubits

Table 1: Comparison between flux-free and flux-dependent analytic frameworks.

In conclusion, the analytic framework developed here not only reproduces established results such as persistent currents and AB phase shifts, but also extends analytic representations to new regimes where topology and flux are central. In the next section, we discuss broader implications, including numerical implementations, interacting systems, and connections to non-Abelian phases in topological quantum computing.

4 DISCUSSION: TOWARD TOPOLOGICAL QUANTUM COMPUTING

The analytic framework developed in this paper has implications that extend beyond mesoscopic physics. By encoding flux-induced quasi-periodicity into analytic functions, the representation provides a platform for exploring topological quantum computation, fault-tolerant architectures, and quantum simulation of interacting systems. In this section, we highlight several directions of future research.

4.1 Numerical Implementations

A practical advantage of analytic representations is the possibility of developing efficient numerical algorithms. In conventional approaches, simulating flux qubits or mesoscopic rings often requires diagonalization of large Hamiltonians or time-evolution under flux-dependent operators. Such methods scale poorly with system size.

The Theta-function framework offers a more compact description. Because quasi-periodicity is built directly into $Q_\phi(z)$, simulations can focus on analytic properties such as reproducing kernels and zero dynamics, bypassing brute-force diagonalization. One can envision hybrid numerical-analytic methods where states are represented by truncated series expansions of $Q_\phi(z)$, and observables are extracted using the kernel (4).

This approach could significantly reduce computational cost and may provide advantages in simulating arrays of flux qubits or larger superconducting circuits, particularly when coupled with modern spectral methods.

4.2 Multi-Particle and Interacting Systems

The analytic representation has so far been applied to single-particle wavefunctions on a circle. An important

extension is to multi-particle and interacting systems on compact topologies.

In such cases, the Hilbert space is spanned by states $q(x_1, \dots, x_N)$ defined on N -fold circles. Introducing a flux Φ threading the system modifies the boundary conditions in each variable, leading to a multi-dimensional quasi-periodic structure.

The corresponding analytic representation would involve multivariable Theta functions, generalizing (2) to higher dimensions. Such an extension would allow the study of flux-dependent entanglement, coherence, and correlation effects.

For example, in a two-particle system one may examine how flux controls the exchange statistics of particles on a compact topology. In interacting systems, flux could modulate collective excitations, potentially giving rise to new topological phases.

4.3 Higher-Genus Surfaces and Modular Structure

The circle is the simplest compact topology. More complex surfaces, such as tori or higher-genus Riemann surfaces, are also relevant in condensed matter and topological field theories.

Theta functions naturally generalize to these cases. Analytic representations on higher-genus surfaces would encode multiple fluxes, each associated with a distinct cycle of the surface. The modular properties of Theta functions then acquire direct physical interpretation: modular transformations correspond to different ways of threading flux through the system.

This line of research could unify analytic representations with the study of modular invariance in conformal field theory and with the use of Riemann surfaces in string theory.

4.4 Connections to Non-Abelian Topological Phases

Perhaps the most exciting extension concerns non-Abelian anyons, which are central to fault-tolerant topological quantum computation [13, 14]. Non-Abelian phases arise in systems where exchanging quasiparticles implements unitary transformations

on the ground-state manifold, providing inherent robustness to local errors.

The analytic framework developed here may serve as a stepping stone toward describing such systems. By encoding flux and topology in analytic functions, one may attempt to generalize the representation to non-Abelian settings, where states transform under higher-dimensional representations of the braid group.

In particular, the monodromy of zeros in the flux-dependent analytic representation suggests an analogy with braiding statistics: as flux is varied, zeros trace structured paths, reminiscent of anyonic worldlines. Investigating this analogy more rigorously may provide analytic tools for understanding non-Abelian braiding in quantum Hall states or Kitaev-type models.

4.5 Future Perspectives

The implications of flux-dependent analytic representations extend across several domains:

- In mesoscopic physics, they provide compact tools for computing persistent currents and interference shifts.
- In quantum technology, they offer analytic perspectives on flux qubits and superconducting circuits.
- In mathematical physics, they connect to modular forms, Riemann surfaces, and the theory of Theta functions.
- In topological quantum computing, they open potential pathways toward analytic descriptions of non-Abelian anyons.

Thus the framework presented here not only addresses immediate questions of flux and boundary conditions, but also lays a foundation for broader exploration of topological and analytic structures in quantum theory.

5 CONCLUSION

In this work we have developed a flux-dependent analytic representation for quantum systems on a circle, based on Jacobi Theta functions. By incorporating the Aharonov–Bohm phase directly into the analytic formalism, we constructed entire analytic wavefunctions that satisfy quasi-periodic boundary conditions. This generalizes the classical Bargmann representation and earlier flux-free Theta function approaches to systems where magnetic flux plays a fundamental role.

We showed that the representation admits a reproducing kernel, ensuring mathematical completeness and preserving Hilbert space structure. The presence of flux modifies the kernel and introduces quasi-periodicity in the analytic domain.

Applications of the framework were demonstrated through explicit calculations. We showed how persistent currents in mesoscopic rings arise naturally from the flux-dependent analytic wavefunctions, and how AB phase shifts in interference devices are directly encoded in the quasi-periodicity of the representation. A systematic comparison with flux-free analytic methods highlighted the advantages of our approach: while flux-free methods capture periodic dynamics, the present framework extends them by including topological effects central to mesoscopic physics and flux qubits.

Beyond immediate applications, we discussed broader implications. Analytic representations provide a promising foundation for numerical implementations, potentially enabling efficient simulations of flux qubits without large-scale Hamiltonian diagonalizations. The formalism also generalizes naturally to multi-particle and interacting systems, where flux can control entanglement and correlation effects. Finally, connections to higher-genus surfaces and non-Abelian topological phases suggest that the analytic representation may contribute to the mathematical foundations of fault-tolerant quantum computing.

Future Directions

Several avenues remain open for further exploration:

- **Numerical Algorithms:** Develop spectral and kernel-based methods for simulating flux-dependent dynamics in mesoscopic devices using truncated analytic expansions.
- **Interacting Systems:** Extend the representation to multi-particle systems on compact manifolds, examining flux-dependent entanglement and collective excitations.
- **Topological Generalizations:** Generalize to higher-genus surfaces and explore the modular properties of Theta functions in quantum systems.
- **Non-Abelian Phases:** Investigate connections between zero dynamics and braiding statistics, with applications to topological quantum computation.

In summary, the flux-dependent Theta function framework unifies analytic methods and topological quantum physics. It provides new mathematical tools, yields physically relevant observables, and opens pathways toward future research in mesoscopic systems, quantum information, and topological quantum computing.

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